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ABSTRACT

This instructor's guide consists of materials for use in teaching a course in geometry designed for students enrolled in postsecondary vocational or technical education programs. Covered in the individual units of the guide are the following topics: basic terms, straight line combinations, angular conversion, circles, polygons, and geometric formulas. Each unit contains some or all of the following components: a unit objective, specific performance objectives, suggested activities for the instructor, a list of references, information sheets, transparency masters, assignment sheets, a test evaluation, and answers to the test. (MN)

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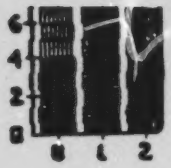

GEOMETRY

Instructor Edition

POST-SECONDARY

VOCATIONAL-TECHNICAL LEVEL

AREA OF VOCATIONAL EDUCATION
COLLEGE OF EDUCATION
UNIVERSITY OF ARKANSAS
FAYETTEVILLE, ARKANSAS 72701

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GEOMETRY

by

FREEMAN D. EADS, Ph.D.

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Developed at the

UNIVERSITY OF ARKANSAS, FAYETTEVILLE

April 1984

in cooperation with the

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ARKANSAS DEPARTMENT OF EDUCATION

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TABLE OF CONTENTS

Unit I	Basic Terms	GEO - 1
Unit II	Straight Line Combinations	GEO - 31
Unit III	Angular Conversion	GEO - 53
Unit IV	Circles	GEO - 89
Unit V	Polygons	GEO - 109
Unit VI	Geometric Formulas	GEO - 141

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This guide was developed from a study of mathematical competencies needed by all post-secondary vocational-technical school students. Study participants included all directors, assistant directors of instruction, and mathematics instructors in the Arkansas vocational-technical schools. Graduates, craft advisory council members, and program instructors from six vocational-technical schools also participated.

Black River	Pulaski
Northwest	Rich Mountain
Pines	White River

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USE OF PUBLICATION

Instructional Units

This guide contains 6 units. Each instructional unit includes some or all of the basic components of a unit of instruction: performance objectives, suggested activities for teachers, information sheets, assignment sheets, visual aids, tests, and answers to the test. Units are planned for more than one lesson or class period of instruction.

Careful study of each instructional unit by the teacher will help to determine:

A. The amount of material that can be covered in each class period

B. The skills which must be demonstrated
   1. Supplies needed
   2. Equipment needed
   3. Amount of practice needed
   4. Amount of class time needed for demonstrations

C. Supplementary materials such as pamphlets or filmstrips that must be ordered

D. Resource people who must be contacted

Objectives

Each unit of instruction is based on performance objectives. These objectives state the goals of the course, thus providing a sense of direction and accomplishment for the student.

Performance objectives are stated in two forms: unit objectives, stating the subject matter to be covered in a unit of instruction; and specific objectives, stating the student performance necessary to reach the unit objective.

Since the objectives of the unit provide direction for the teaching-learning process, it is important for the teacher and students to have a common understanding of the intent of the objectives. A limited number of performance terms have been used in the objectives for this curriculum to assist in promoting the effectiveness of the communication among all individuals using the materials.

Reading of the objectives by the student should be followed by a class discussion to answer any questions concerning performance requirements for each instructional unit.

Teachers should feel free to add objectives which will fit the material to the needs of the students and community. When teachers add objectives, they should remember to supply the needed information, assignment and/or job sheets, and criterion tests.

Suggested Activities for the Instructors

Each unit of instruction has a suggested activities sheet outlining steps to follow in accomplishing specific objectives. Duties of instructors will vary according to the particular unit; however, for best use of the material they should include the following: provide students with objective sheet, information sheet, and assignment sheets; make transparencies and arrange for resource materials and people; discuss unit and specific objectives and information sheet; give test. Teachers are encouraged to use any additional instructional activities and teaching methods to aid students in accomplishing the objectives.

Information Sheets

Information sheets provide content essential for meeting the cognitive(knowledge) objectives in the unit. The teacher will find that the information sheets serve as an excellent guide for presenting the background knowledge necessary to develop the skill specified in the unit objective.

Students should read the information sheets before the information is discussed in class. Students may take additional notes on the information sheets.

Transparency Masters

Transparency masters provide information in a special way. The students may see as well as hear the materials being presented, thus reinforcing the learning process. Transparencies may present new information or they may reinforce information presented in tbe information sheets. They are particularly effective when identification is necessary.

Transparencies should be made and placed in the notebook where they will be immediately available for use. Transparencies direct the class's attention to the topic of discussion. They should be left on the screen only when topics shown are under discussion.

Assignment Sheets

Assignment sheets give direction to study and furnish practice for paper and pencil activities to develop the knowledges which are necessary prerequisites to skill development. These may be given to the student for completion in class or used for homework assignments. Answer sheets are provided which may be used by the student and/or teacher for checking student progress.

Test Evaluation

Paper-pencil and performance tests have been constructed to measure student achievement of each objective listed in the unit of instruction. Individual test items may be pulled out and used as a short test to determine student achievement of a particular objective. This kind of testing may be used as a daily quiz and will help the teacher spot difficulties being encountered by students in their efforts to accomplish the unit objective. Test items for objectives added by the teacher should be constructed and added to the test.

Test Answers

Test answers are provided for each unit. These may be used by the teacher and/or student for checking student achievement of the objectives.

BASIC TERMS
UNIT I

UNIT OBJECTIVE

After completion of this unit, the student should be able to state the definition of given basic geometric terms. The student should also be able to recall relationships that follow given geometric combinations. This knowledge will be evidenced by memorizing the definitions and relationships presented in the information sheets and by scoring 85 percent on the unit test.

SPECIFIC OBJECTIVES

After completion of this unit of instruction, the student will be able to:

1. Give the definition normally accepted in Euclidian or Plane Geometry for straight and curved lines, points, and angles.
2. Describe the main kinds of angles.
3. Define the three main methods of angular measurement
   a. Define a right angle in degrees.
   b. Define a right angle in radians.
   c. Define a right angle in grads.
4. Interpret given symbols common to plane geometry.

BASIC TERMS
UNIT I

SUGGESTED ACTIVITIES

I. Make transparencies.

II. Provide students with objective sheet.

III. Provide students with information sheets.

IV. Discuss unit and specific objectives.

V. Discuss information sheets.

A. Use transparencies during discussion.

B. Use chalkboard during discussion.

VI. Pass out and discuss assignment sheets.

VII. Assist students as they work through assignment sheets.

VIII. Give test.

IX. Evaluate and record test results.

INSTRUCTIONAL MATERIALS

I. Included in the unit:

A. Objective Sheet

B. Information Sheet

C. Transparency Masters

1. TM#1--Angles

2. TM#2--Adjacent Angles

3. TM#3--Obtuse, acute, right and vertical angles

D. Assignment Sheets

1. Assignment Sheet #1--Drawing acute, obtuse, right and adjacent angles, label vertices, complementary and supplementary angles

2. Assignment Sheet #2--Definitions of angles in terms of degrees, grads and rads

3. Assignment Sheet #3--Symbols

E. Answers to Assignment Sheets

F. Test

G. Answers to Test

II. References

A. Cleaves, Cheryl, Margie Hobbs, and Paul Dudenhefer. Basic Mathematics for Trades and Technologies, Englewood Cliffs, NJ: Prentice Hall, Incorporated, 1983.

B. Smith, Robert D. Vocational-Technical Mathematics Albany, NY: Delmar Publishers Incorporated, 1983.

ANGLES

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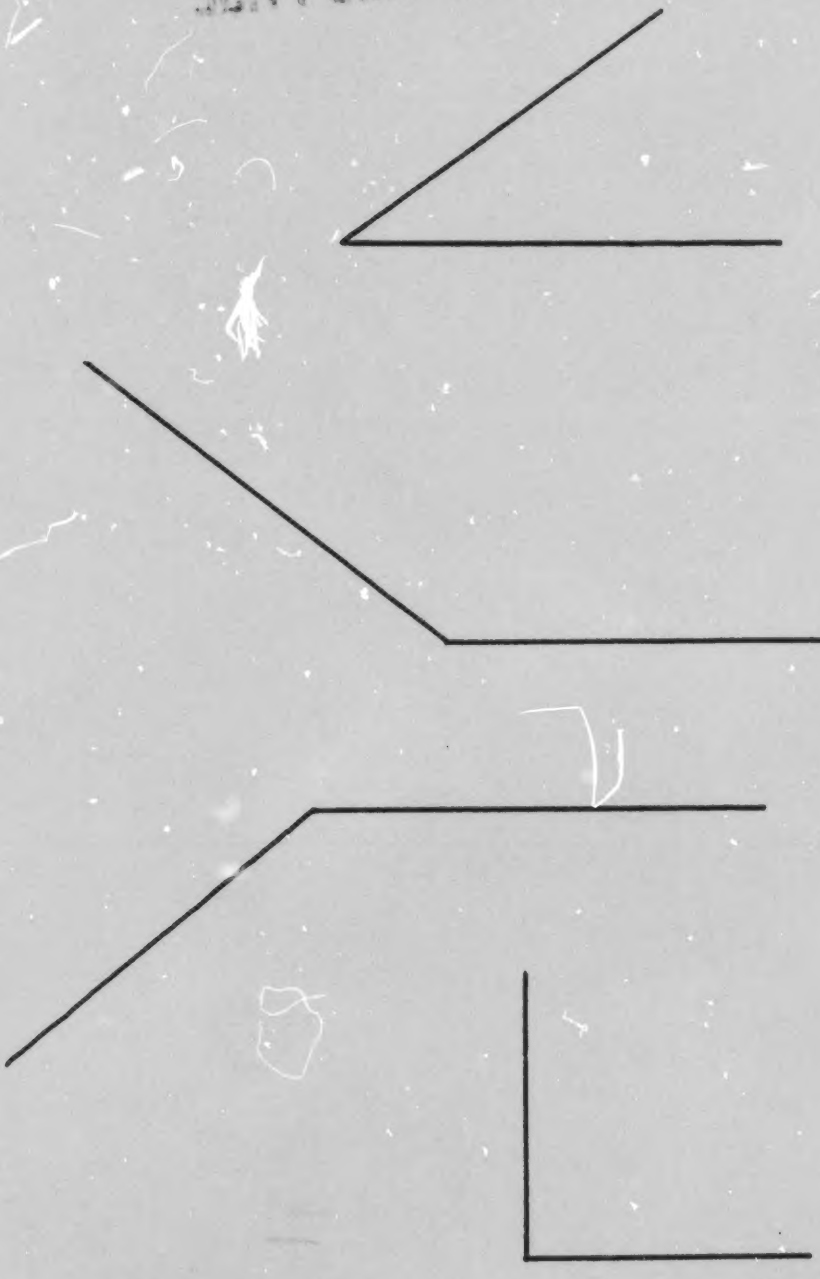
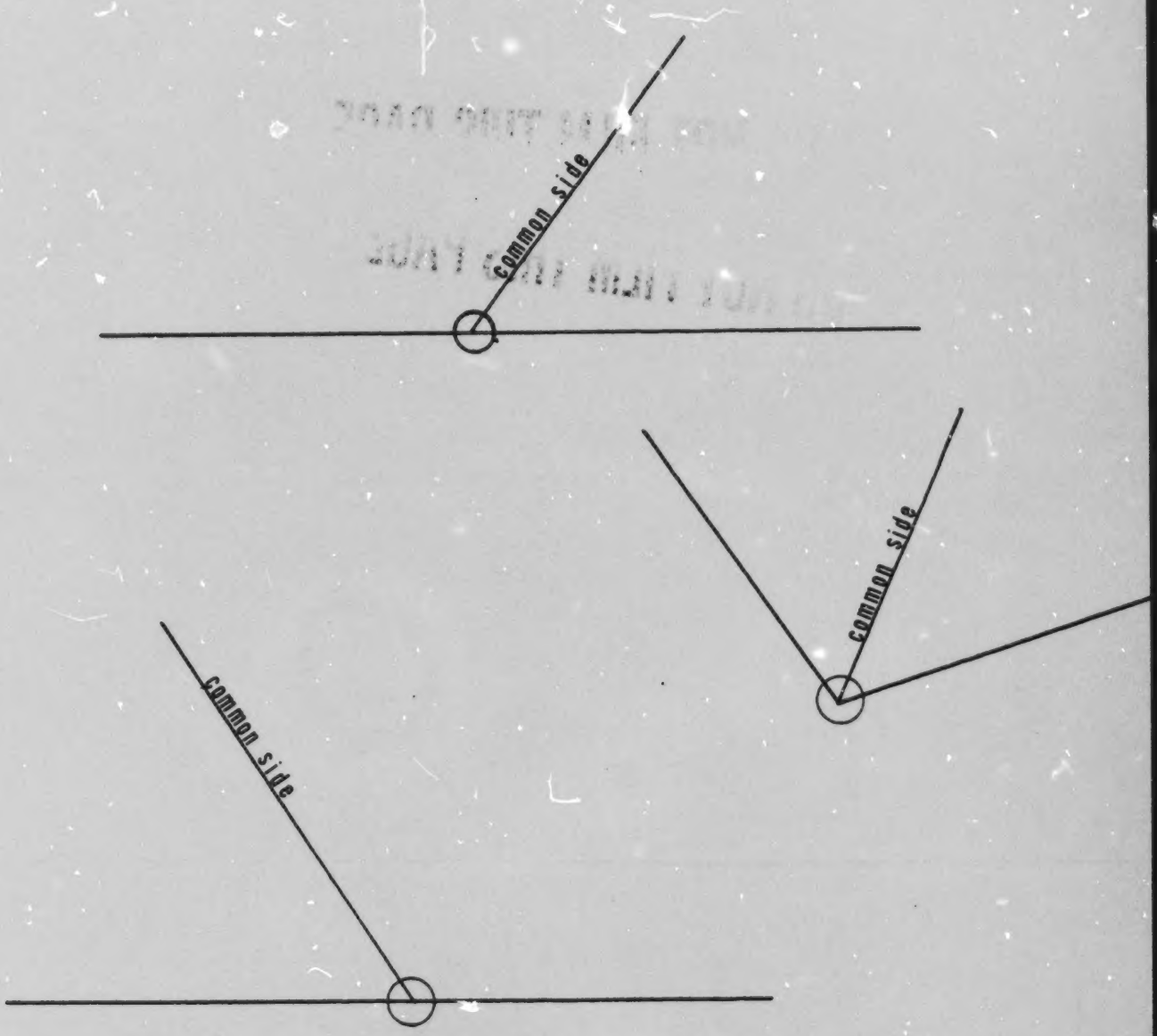


TM#1

ADJACENT ANGLES



(NOTE: "Common" means shared as far as this transparency is concerned.)

TM#2

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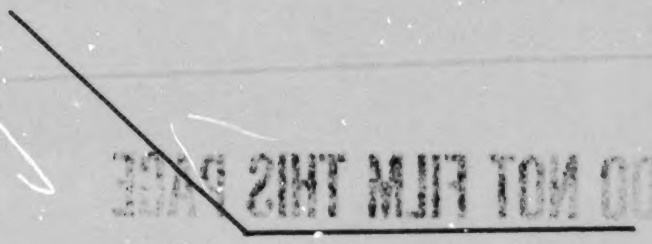
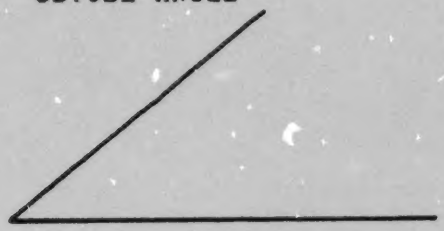
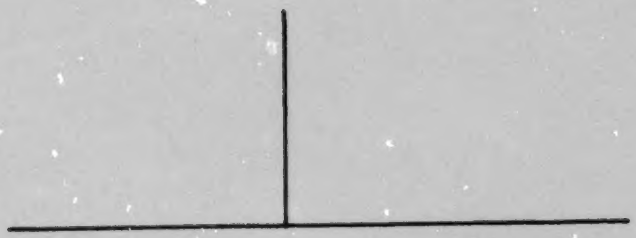


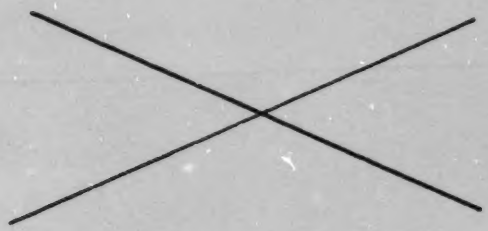
OBTUSE ANGLE



ACUTE ANGLE



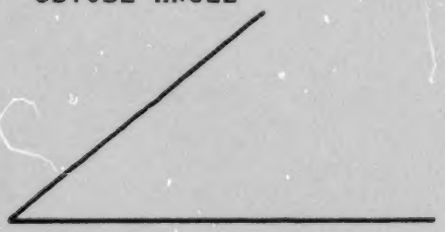
RIGHT ANGLE



VERTICAL ANGLES

TM#3

OBTUSE ANGLE



ACUTE ANGLE

RIGHT ANGLE

VERTICAL ANGLES

BASIC TERMS
UNIT I

INFORMATION SHEET

I. Terms and Definitions

A. Line--The boundary of a surface or the intersection of 2 or more surfaces

1. Straight Line--A line which does not change direction

(NOTE: When the word "line" is used, it can be assumed that it is "straight".)

2. Curved Line--A line which DOES change direction; it may or may not change at a constant rate

(NOTE: Lines do not have any dimension except length. They do NOT have width or thickness.)

B. Point--A point has only the characteristic of position; it does NOT have length, width or thickness

(NOTE: Points are generated or caused by the intersection of 2 or more lines or sometimes by the same line if that line is curved, as is the case of an ampersand, "&", in which 2 points are generated by a single curved line intersecting with itself.)

(NOTE: Points may also determine the end or beginning of a line segment.)

Example:

A_____B
Line Segment AB

(NOTE: Lines or points cannot be seen. When a line is drawn on a piece of paper; for example, AB above, so-called line segment AB, in addition to having the required characteristic of length, also has an additional dimension--width--(about 0.005")--so it is no longer a true line. But as long as it is agreed that A_____B is a symbol representing line segment AB which really cannot be seen, everything works out fine.)

C. Angle--Angles are formed by two straight lines intersecting at a given point; the lines forming the

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angle are called "sides"

D. Vertex--The point where the two lines forming an angle meet; two or more "vertexes" are really called "vertices"

E. Adjacent Angles--Angles sharing a common vertex and a common side

F. Degree (°)--A unit of angular measurement; 360° equal one revolution or a complete circle

G. Minute of Angle-(')--1/60th of a degree

H. Second of Angle--(")--1/60th of a minute; 1/3600th of a degree

I. Radian--(Rad or Rads)--Another method of angular measurement

(NOTE: 2π rad = 360°

$$1 \text{ rad} = \frac{360°}{2\pi}$$

$$1 \text{ rad} = \frac{360°}{2(3.1416)}$$

1 rad = 57.296° or 57°17'44.8"

(NOTE: Radians are NOT normally used in Plane Geometry. They ARE used in problems involving "angular displacement" or rotation where degrees and revolutions are not so handy.)

J. Grads--one Grad = 1/400th of a circle or 400 grads = 360°

(NOTE: The best way to think of a grad is 1/100th of a right angle. Grads are a fairly recent attempt at curing problems caused by the degree-minute-second method. They are not very popular yet--some dictionairies do not include them, but many calculators have keys that will convert from degrees-grads-radians; therefore, you should be aware that "they" are out there.)

K. Right Angle--An angle equal to 1/4 of a revolution, 90°, $\pi/2$ rads, or 100 grads

L. Acute Angle--An angle whose measure is less than a right angle (90°)

M. Obtuse Angle--An angle greater than 90° but less than 180°

N. Straight Angle--An angle whose measure is equal to 180° (or π rads or 200 grads)

(NOTE: By definition ALL straight lines are straight angles.)

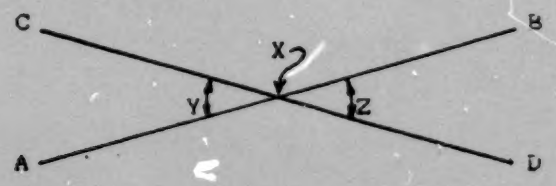
O. Reflex Angle--An angle greater than 180° but less than 360°

P. Complementary Angles--Two angles are said to be complementary if their sums are equal to 90°

Q. Supplementary Angles--Two angles are said to be supplementary if their sums are equal to 180°

R. Vertical Angles--Vertical angles are angles sharing the same sides, but occuring on opposite sides of the vertex

Example:



(NOTE: Lines AB and CD intersect at point X; angles Y and Z are vertical angles.)

(NOTE: An interesting and important characteristic of vertical angles is that they are equal: thus $\angle y = \angle z$.)

II. Geometric Symbology

A. $\angle$ and $\measuredangle$ are symbols for angles

B. $\llcorner$ and $\llcorner$ are symbols for right angles

C. $\measuredangle s$ or $\angle s$ means more than one angle

D. $\overline{AB}$ means line segment AB

E. $\overset{\frown}{AB}$ means arc AB

F. Greek letters commonly used in math and geometry

a. α --alpha (lower case)

b. β --beta (lower case)

c. θ --theta

d. φ --phi

e. π --pi

f. Σ --sigma (upper case);
σ --sigma (lower case)

G. Other symbols

a. ⊥ --perpendicular to

b. △ --triangle

c. ◺ --right triangle

d. ⊙ --circle

e. ≠ --does not equal

f. ∴ --therefore

BASIC TERMS
UNIT 1

ASSIGNMENT SHEET #1--DRAWING ANGLES

1. Draw an acute angle. Draw a circle around the vertex.

2. Draw an obtuse angle. Draw a circle around the vertex.

3. Draw a right angle. Extend the sides. How many right angles are formed? How many pairs of adjacent angles are formed?

4. Draw a pair of adjacent complementary angles. Put the letter "A" at the vertex.

5. Draw a pair of adjacent supplementary angles. Put the letter "X" at the vertex.

6. Draw at least two vertical angles. Put the letter "Z" at their common vertex. Is it possible to draw just two vertical angles?

BASIC TERMS
UNIT I

ASSIGNMENT SHEET #1--DRAWING ANGLES

1. Draw an acute angle. Draw a circle around the vertex.

2. Draw an obtuse angle. Draw a circle around the vertex.

3. Draw a right angle. Extend the sides. How many right angles are formed? How many pairs of adjacent angles are formed?

4. Draw a pair of adjacent complementary angles. Put the letter "A" at the vertex.

5. Draw a pair of adjacent supplementary angles. Put the letter "X" at the vertex.

6. Draw at least two vertical angles. Put the letter "Z" at their common vertex. Is it possible to draw just two vertical angles?

BASIC TERMS
UNIT I

ASSIGNMENT SHEET #2--DEFINING ANGLES

Directions: Fill in the blanks with the correct number or word.

1. There are _____ degrees in a right angle.
2. There are _____ degrees in one revolution.
3. A _____ angle represents 1/4 of a revolution.
4. There are _____ grads in a right angle.
5. There are _____ grads in a straight angle.
6. There are _____ grads in a revolution.
7. There are _____ π radians in one revolution.
8. There are _____ π radians in a straight angle.
9. There are _____ π radians in 1/4 of a revolution.
10. _____ rads = _____ ° = _____ grads = a right angle.

BASIC TERMS
UNIT 1

ASSIGNMENT SHEET #3--SYMBOLS

Directions: Translate the following messages into plain English. (numbers 1 - 4)

1. $\angle ABC \neq 90°$
   $\angle ABC < 90°$
   $\therefore \angle ABC < 180°$

2. $\angle \theta + \angle \alpha = 180°$
   $\angle \theta = \angle \alpha$
   $\therefore \angle \theta = 90°$
   $\angle \alpha = 90°$

3. $m\angle ABC = 90°$
   $\therefore m\angle ABC = \pi/2$ rads

4. $\angle \alpha + \angle \beta = 180°$
   $\angle \alpha > \angle \beta$

5. Referring to question 4.

   $\angle \alpha$ and $\angle \beta$ are _____ (supplementary, complementary)

   Draw and label angles that would satisfy the given conditions.

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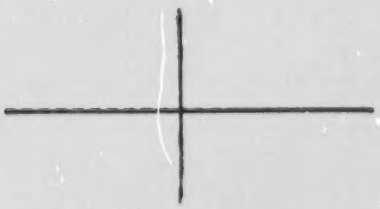
BASIC TERMS
UNIT I

ANSWERS TO ASSIGNMENT SHEETS

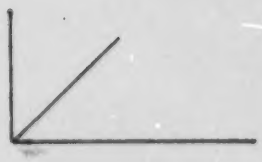
ASSIGNMENT SHEET #1

1. Any angle less than 90° is acceptable.

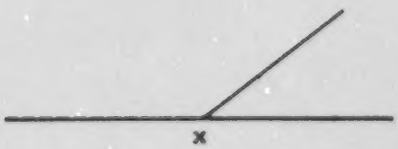
2. Any angle between 90° and 180° is acceptable.

3. 

4 right angles are formed
4 pairs of adjacent angles are formed

4. 

Criteria: Exterior sides of angle must be perpendicular; a third line must be between the exterior sides

5. 

The exterior sides must be a single straight line the third line may be anywhere between the two

6.

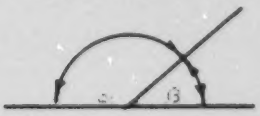
No. Four vertical angles are always formed by the intersection of two lines

ASSIGNMENT SHEET #2

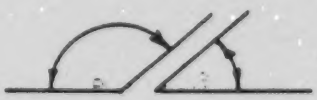
1. 90
2. 360
3. right
4. 100
5. 200
6. 400
7. 2
8. 1 (π radians)
9. 1/2 (π radians)
10. π/2 rads = 90° = 100 grads = a right angle

ASSIGNMENT SHEET #3

1. Angle ABC does not equal ninety degrees. Angle ABC is less than ninety degrees. Therefore, angle ABC does not equal one hundred eighty degrees.
2. Angle theta plus angle alpha equal one hundred eighty degrees. Angle theta equals angle alpha. Therefore, angle theta equals ninety degrees and angle alpha equals ninety degrees.
3. Right angle ABC equals ninety degrees. Therefore, right angle ABC equals pi over two radians.
4. Angle alpha plus angle beta equal one hundred eighty degrees. Angle alpha is greater than angle beta.
5. Supplementary



or



Criteria: If drawn as adjacent ∠s, exterior side must be a straight angle, common side may be anywhere. Any of

the resulting adjacent angles may be labeled as or
.

If not drawn as adjacent angles, an honest attempt to make the sides parallel as shown should have been made.

A third possibility of angles disoriented from one another should be allowed if the student can demonstrate that their sum would equal 180 degrees--perhaps by actually cutting the angles out and placing them adjacent to one another along a straight edge. This would illustrate the important concept that geometric figures may be moved without changing their properties.

BASIC TERMS

UNIT I

TEST

Directions: In the left hand column is a list of terms. Below the list is a column which contains descriptive phrases or synonymous terms. Match the descriptions to the terms by placing the number or numbers of the description(s) in the blank at the left of the appropriate term.

____ A. Line
____ B. Point
____ C. Angle
____ D. Vertex
____ E. Curved Line
____ F. Straight Line

1. the point where the two lines forming an angle meet

2. the boundary of a surface or the intersection of 2 or more surfaces

3. a geometric entity which has the characteristic of length, but not those of width or thickness

4. a geometric entity which may be the boundry of a surface or the intersection of 2 or more surfaces AND which does NOT change direction

5. a geometric entity which may be the boundary of a surface or the intersection of 2 or more surfaces AND which DOES change direction

6. a geometric entity which has only the characteristic of position and does NOT have those of length, width and thickness

7. a geometric entity formed by two straight lines intersecting at a given point

2. Complete the following statement by filling in the blank with the correct word, symbol, or number.

a. A right angle is equal to ___ of a revolution.

b. A right angle is equal to ___ degrees.

c. An acute angle is ___ than a right angle.

d. An obtuse angle is ___ than a right angle.

e. A straight angle is equal to___ degrees.

f. A reflex angle is greater than ___ degrees, but less than 360 degrees.

g. Two angles are said to be complementary if their sums are equal to ___ degrees.

h. Two angles are said to be supplementary if their sums are equal to ___ degrees.

i. Two angles which share the same sides, but occur on opposite sides of the vertex are called ___ angles.

j. Angles sharing a common vertex and a common side are called ___ angles.

3. Fill in the blanks with the correct word, symbol, or number.

a. There are ___ degrees in a right angle.

b. A ___ angle represents 1/4 of a revolution.

c. There are ___ grads in a right angle.

d. There are ___ grads in a straight angle.

e. There are ___ grads in a revolution.

f. There are ___ radians in one revolution.

g. There are ___ radians in a straight angle.

h. There are ___ radians in 1/4 of a revolution.

4. Translate the following statements into plain English:

a. $\angle ABC \neq 90°$ _____

b. $\angle ABC < 90°$ _____

c. $\therefore \angle ABC \neq 180°$ _____

d. $\angle \alpha + \angle \theta = 180°$ _____

e. $\angle \theta = \angle \alpha$ _____

BASIC TERMS

UNIT I

TEST

Directions: In the left hand column is a list of terms. Below the list is a column which contains descriptive phrases or synonymous terms. Match the descriptions to the terms by placing the number or numbers of the description(s) in the blank at the left of the appropriate term.

____ A. Line
____ B. Point
____ C. Angle
____ D. Vertex
____ E. Curved Line
____ F. Straight Line

1. the point where the two lines forming an angle meet

2. the boundary of a surface or the intersection of 2 or more surfaces

3. a geometric entity which has the characteristic of length, but not those of width or thickness

4. a geometric entity which may be the boundry of a surface or the intersection of 2 or more surfaces AND which does NOT change direction

5. a geometric entity which may be the boundary of a surface or the intersection of 2 or more surfaces AND which DOES change direction

6. a geometric entity which has only the characteristic of position and does NOT have those of length, width and thickness

7. a geometric entity formed by two straight lines intersecting at a given point

2. Complete the following statement by filling in the blank with the correct word, symbol, or number.

a. A right angle is equal to ___ of a revolution.

b. A right angle is equal to ___ degrees.

c. An acute angle is ___ than a right angle.

d. An obtuse angle is ___ than a right angle.

e. A straight angle is equal to___ degrees.

f. A reflex angle is greater than ___ degrees, but less than 360 degrees.

g. Two angles are said to be complementary if their sums are equal to ___ degrees.

h. Two angles are said to be supplementary if their sums are equal to ___ degrees.

i. Two angles which share the same sides, but occur on opposite sides of the vertex are called ___ angles.

j. Angles sharing a common vertex and a common side are called ___ angles.

3. Fill in the blanks with the correct word, symbol, or number.

a. There are ___ degrees in a right angle.

b. A ___ angle represents 1/4 of a revolution.

c. There are ___ grads in a right angle.

d. There are ___ grads in a straight angle.

e. There are ___ grads in a revolution.

f. There are ___ radians in one revolution.

g. There are ___ radians in a straight angle.

h. There are ___ radians in 1/4 of a revolution.

4. Translate the following statements into plain English:

a. $\angle ABC \neq 90°$ _____

b. $\angle ABC < 90°$ _____

c. $\angle ABC \not\equiv 180°$ _____

d. $\angle \alpha + \angle \theta = 180°$ _____

e. $\angle \theta = \angle \alpha$ _____

f. $\therefore \angle \circledcirc = 90°$ _____

and $\angle \circ = 90°$ _____

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BASIC TERMS
UNIT I

ANSWERS TO TEST

1. a. 2; 3
   b. 6
   c. 7
   d. 1
   e. 5
   f. 4

2. a. 1/4
   b. 90
   c. less
   d. greater
   e. 180
   f. 180
   g. 90
   h. 180
   i. vertical
   j. adjacent

3. a. 90
   b. right
   c. 100
   d. 200
   e. 400
   f. 2π
   g. π
   h. $\pi/2$

4. a. Angle ABC is not equal to ninety-degrees
   b. Angle ABC is less than ninety-degrees.
   c. Therefore, angle ABC does not equal 180 degrees.
   d. Angle alpha plus angle theta equal one hundred eighty degrees.
   e. Angle theta equals angle alpha.
   f. Therefore, angle theta equals ninety degrees (and) angle alpha equals ninety degrees.

STRAIGHT LINE COMBINATIONS

UNIT II

UNIT OBJECTIVE

After completion of this unit, the student should be familiar with straight lines on a plane surface. The student should also be able to recall certain basic relationships and terms relating to straight line combinations. This knowledge will be evidenced by memorizing the definitions and relationships presented in the information sheets and by scoring 85 percent on the unit test.

SPECIFIC OBJECTIVES

After completion of this unit of instruction, the student will be able to:

1. Draw oblique, parallel and perpendicular lines
2. Draw a transversal intersecting oblique, parallel and/or perpendicular lines
3. Identify the angles resulting from the intersection of the transversal with said lines
4. Determine which angles are equal when parallel lines are cut by a transversal

STRAIGHT LINE COMBINATIONS

UNIT II

SUGGESTED ACTIVITIES

I. Provide students with objective sheet.

II. Provide students with information sheets.

III. Discuss unit and specific objectives.

IV. Discuss information sheets.

V. Discuss assignment sheets.

VI. Assist students as they work through assignment sheets.

VII. Give test.

VIII. Evaluate and record test results.

INSTRUCTIONAL MATERIALS

I. Included in the unit:

A. Objective Sheet

B. Information Sheet

C. Assignment Sheets

1. Assignment Sheet #1--Draw parallel, coincidental oblique and perpendicular lines

2. Assignment Sheet #2--Draw oblique and parallel line cut by transversal; label interior/exterior angles

3. Assignment Sheet #3--Corresponding angles

D. Answers to Assignment Sheets

E. Test

F. Answers to Test

II. References

A. Cleaves, Cheryl, Margie Hobbs, and Paul Dudenhefer. Basic Mathematics for Trades and Technologies, Englewood Cliffs, NJ: Prentice Hall, Incorporated, 1983.

B. Smith, Robert D. Vocational-Technical Mathematics Albany, NY: Delmar Publishers Incorporated, 1983.

STRAIGHT LINE COMBINATIONS

UNIT II

INFORMATION SHEET

I. Terms and Definitions:

(NOTE: If you draw two straight lines on a plane surface (a piece of paper, for example) and if these straight lines are extended forever and ever, one of three things is going to happen. They will either intersect, or cross; they will never cross; or they will coincide (be on top of one another).

A. Parallel lines--lines which will never cross; two or more lines are parallel if they are equidistant throughout their lengths or two or more lines are parallel if they will never intersect, even though extended into infinity

(NOTE: Either definition is acceptable.)

B. Intersecting line--two straight lines which intersect, will intersect at one point, and one point only; there are two classes of intersect lines--oblique and perpendicular

C. Oblique lines--lines which change their distance and would intersect if extended on the side of least distance; the adjacent angles formed at their intersection are unequal

D. Perpendicular lines--two lines are perpendicular if they form equal adjacent angles at their point of intersection; or said another way, if they form adjacent right angles

(NOTE: Lines which intersect form vertical angles as well as adjacent angles; opposite or vertical angles are equal. [Review information sheet for Unit I, Item I:R])

E. Transversal--a line which intersects (or cuts) two or more lines

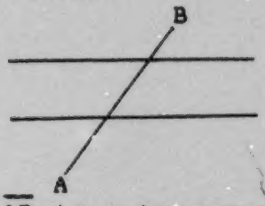


Fig. 1

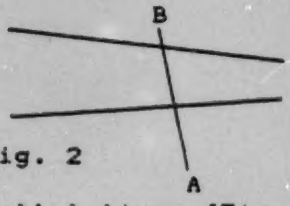
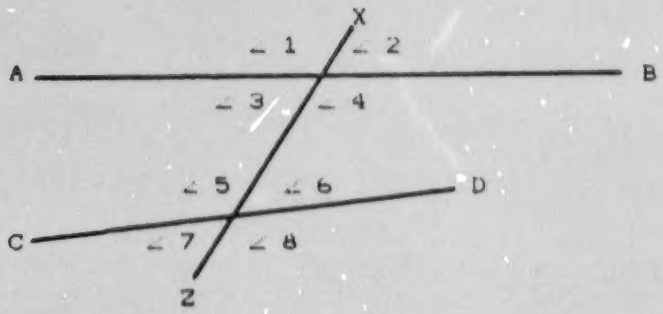


Fig. 2

(NOTE: $\overline{AB}$ is a transversal cutting parallel lines [Fig. 1] and oblique lines [Fig. 2].)



(NOTE: In Fig. 3, we have two lines, $\overline{AB}$ and $\overline{CD}$, cut by a transversal, $\overline{XZ}$. A total of 8 angles are formed. The angles located on the outside of the lines (∠ 1 and ∠ 2; ∠ 7 and ∠ 8) are called exterior angles. The angles betwen the lines (∠ 3 and ∠ 4; ∠ 5 and ∠ 6) are called interior angles. If the angles are on the same side of their respective lines (∠ 1 and ∠ 5 for example), they are called corresponding angles. Other pairs of corrspcnding angles would be: ∠ 3 and ∠ 7; ∠ 2 and ∠ 6; ∠ 4 and ∠ 8.
Alternate interior angles are interior angles on opposite sides of the transversal and each has its own vertex. Alternate exterior angles are exterior angles on opposite sides of the transaversal and each has its own vertex. Alternate interior angles are not adjacent. Alternate exterior angles angles are not adjacent. By definition, alternate interior angles must have separate vertices so they could not possible be adjacent. The same is true for opposite exterior angles also.)

(NOTE: If we talk about parallel lines being cut by a transversal, then several important "theorems" and "corollaries" apply. [A theorem is a "provable fact" and a corollary is a provable fact based on a theorem.] For the sake of clarity, theorems and corollaries will be referred to as "Rules".)

II. Rules

Rule 1--If parallel lines are cut by a transversal, the alternate interior angles are equal.

Rule 2--If parallel lines are cut by a transversal, the alternate exterior angles are equal.

Rule 3--If two parallel lines are cut by a transversal, the corresponding angles are equal.

(NOTE: These rules can also be turned around as follows:)

Rule 4--If two lines are cut by a transversal and if the alternate interior angles are equal, the lines are parallel.

STRAIGHT LINE COMBINATIONS

UNIT II

INFORMATION SHEET

I. Terms and Definitions:

(NOTE: If you draw two straight lines on a plane surface (a piece of paper, for example) and if these straight lines are extended forever and ever, one of three things is going to happen. They will either intersect, or cross; they will never cross; or they will coincide (be on top of one another).

A. Parallel lines--lines which will never cross; two or more lines are parallel if they are equidistant throughout their lengths or two or more lines are parallel if they will never intersect, even though extended into infinity

(NOTE: Either definition is acceptable.)

B. Intersecting line--two straight lines which intersect, will intersect at one point, and one point only; there are two classes of intersect lines--oblique and perpendicular

C. Oblique lines--lines which change their distance and would intersect if extended on the side of least distance; the adjacent angles formed at their intersection are unequal

D. Perpendicular lines--two lines are perpendicular if they form equal adjacent angles at their point of intersection; or said another way, if they form adjacent right angles

(NOTE: Lines which intersect form vertical angles as well as adjacent angles; opposite or vertical angles are equal. [Review information sheet for Unit I, Item I:R])

E. Transversal--a line which intersects (or cuts) two or more lines

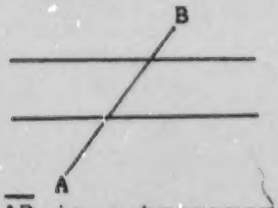


Fig. 1

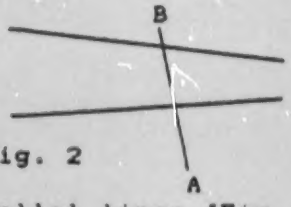
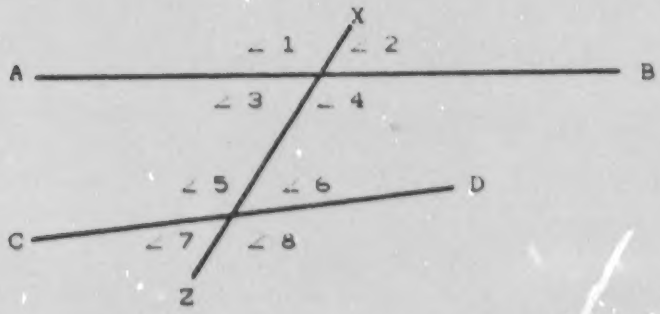


Fig. 2

(NOTE: $\overline{AB}$ is a transversal cutting parallel lines [Fig. 1] and oblique lines [Fig. 2].)



(NOTE: In Fig. 3, we have two lines, $\overline{AB}$ and $\overline{CD}$, cut by a transversal, $\overline{XZ}$. A total of 8 angles are formed. The angles located on the outside of the lines (∠ 1 and ∠ 2; ∠ 7 and ∠ 8) are called exterior angles. The angles betwen the lines (∠ 3 and ∠ 4; ∠ 5 and ∠ 6) are called interior angles. If the angles are on the same side of their respective lines (∠ 1 and ∠ 5 for example), they are called corresponding angles. Other pairs of corrsponding angles would be: ∠ 3 and ∠ 7; ∠ 2 and ∠ 6; ∠ 4 and ∠ 8.
Alternate interior angles are interior angles on opposite sides of the transversal and each has its own vertex. Alternate exterior angles are exterior angles on opposite sides of the transversal and each has its own vertex. Alternate interior angles are not adjacent. Alternate exterior angles angles are not adjacent. By definition, alternate interior angles must have separate vertices so they could not possible be adjacent. The same is true for opposite exterior angles also.)

(NOTE: If we talk about parallel lines being cut by a transversal, then several important "theorems" and "corollaries" apply. [A theorem is a "provable fact" and a corollary is a provable fact based on a theorem.] For the sake of clarity, theorems and corollaries will be referred to as "Rules".)

II. Rules

Rule 1--If parallel lines are cut by a transversal, the alternate interior angles are equal.

Rule 2--If parallel lines are cut by a transversal, the alternate exterior angles are equal.

Rule 3--If two parallel lines are cut by a transversal, the corresponding angles are equal.

(NOTE: These rules can also be turned around as follows:)

Rule 4--If two lines are cut by a transversal and if the alternate interior angles are equal, the lines are parallel.

Rule 5--If two lines are cut by a transversal and if the alternate exterior angles are equal, the lines are parallel.

Rule 6--If two lines are cut by a transversal, and the corresponding angles are equal, the lines are parallel.

STRAIGHT LINE COMBINATIONS

UNIT II

ASSIGNMENT SHEET #1--PARALLEL, COINCIDENTAL OBLIQUE AND PERPENDICULAR LINES

1. Draw a pair of parallel lines

2. Draw 3 parallel lines

3. Draw a line $\overline{AB}$ which will coincide with line $\overline{YZ}$.

Y _____ Z

4. Draw a pair of obliques lines intersecting at pcint "X".

5. Draw a line $\overline{AB}$ perpendicular to line $\overline{CD}$ and intersecting at point X.

C _____ D
X

STRAIGHT LINE COMBINATIONS

UNIT II

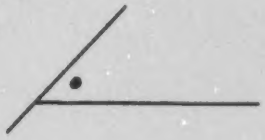
ASSIGNMENT SHEET #2--OBLIQUE AND PARALLEL LINE CUT BY TRANSVERSAL; INTERIOR AND EXTERIOR ANGLES

1. Draw a pair of obliques lines cut by a transversal. Label the transversal $\overline{AB}$.

2. Label the four exterior angles $\angle E_1$; $\angle E_2$; $\angle E_3$; and $\angle E_4$

3. Label the four interior angles $\angle I_1$; $\angle I_2$; $\angle I_3$; and $\angle I_4$

4. Draw a pair of parallel lines cut by a transversal, $\overline{YZ}$. Label any one exterior angle $\angle \odot$. Indicate all of the angles equal to $\angle \odot$ by making a big dot near the vertex of each angle like this:



STRAIGHT LINE COMBINATIONS

UNIT II

ASSIGNMENT SHEET #8 - CORRESPONDING ANGLES

1. Draw two lines cut by a transversal $\overline{XZ}$. Label any pair of corresponding angles, A.

2. Label any other pair of corresponding angles B.

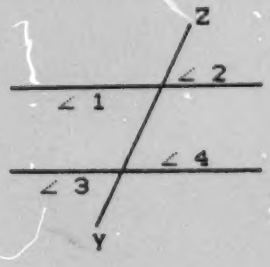
3. Label any other unmarked pair of corresponding angles C.

4. Label any other unmarked pair of corresponding angles D.

5. Based on <u>your</u> drawing, are the two angles marked "C" equal? Why or why not?

6. Given: Lines $\overline{AB}$ and $\overline{CD}$ cut by a transversal $\overline{YZ}$
   $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

   Is this sufficient information to prove them parallel?



STRAIGHT LINE COMBINATIONS

UNIT II

ASSIGNMENT SHEET #3--CORRRESPONDING ANGLES

1. Draw two lines cut by a transversal $\overline{XZ}$. Label any pair of corresponding angles, A.

2. Label any other pair of corresponding angles B.

3. Label any other unmarked pair of corresponding angles C.

4. Label any other unmarked pair of corresponding angles D.

5. Based on <u>your</u> drawing, are the two angles marked "C" equal? Why or why not?

6. Given: Lines $\overline{AB}$ and $\overline{CD}$ cut by a transversal $\overline{YZ}$
   $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

   Is this sufficient information to prove them parallel?

STRAIGHT LINE COMBINATIONS

UNIT II

ANSWERS TO ASSIGNMENT SHEETS

Assignment Sheet #1

1. _____ (lines do not have to be horizontal)

   _____

2. _____ (lines do not have to be horizontal nor equidistant)

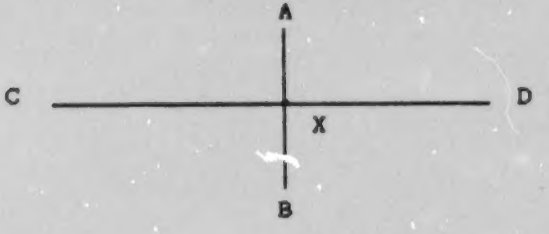
   _____

   _____

3. A Y _____ Z B

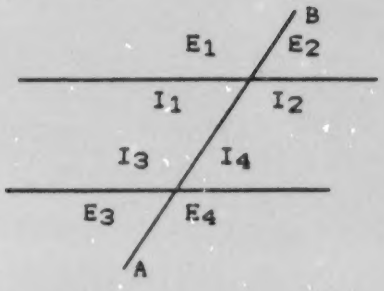
4.

5.

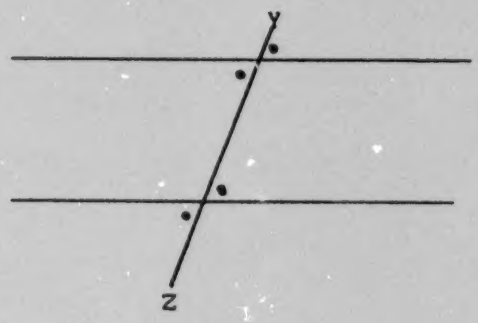


Assignment Sheet #2

1., 2., and 3. (NOTE: Lines may, or may not, be parallel.)



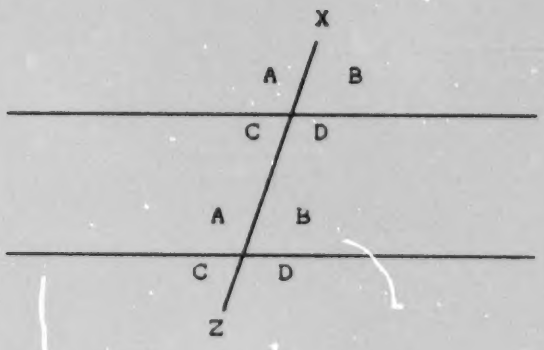
4.



or

Assignment Sheet #3

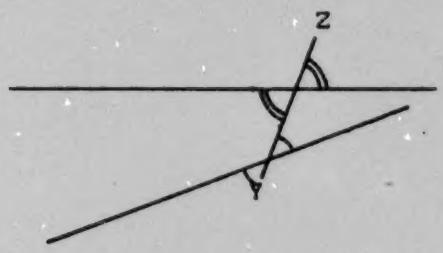
1. through 4.



5. If lines were drawn parallel answer is yes because rule 3 says, "If parallel lines are cut by a transversal, the corresponding angles are equal."

If lines were drawn as oblique lines the answer is no. If the angles were equal the lines would be parallel and that is not the case.

6. No. $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ by virtue of their being vertical angles. This would be the case no matter how oblique they might be.

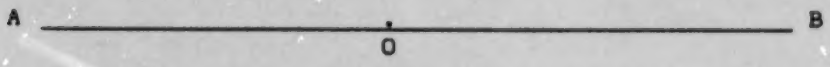


STRAIGHT LINE COMBINATIONS

UNIT II

TEST

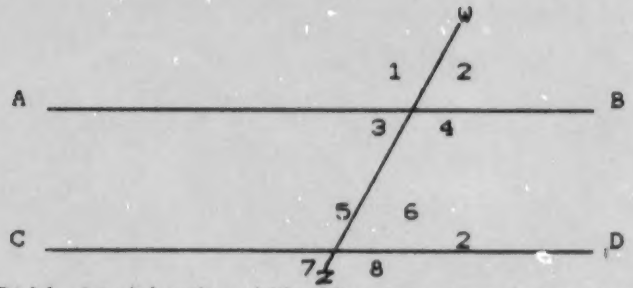
1. Draw a line $\overline{ZZ}$ perpendicular to line $\overline{AB}$ and passing through point "O".



2. Label two adjacent angles ⊙ and ⌀.

Does $\angle$ ⊙ = $\angle$ ⌀?

3. Given: lines AB and CD are parallel lines and line WZ is a transversal cutting lines AB and CD, $\angle$ 1 =/= 90°



3. Fill in blank with the appropriate word from the following list:

3. Fill in blank with the appropriate word from the following list:

interior
exterior
vertical

∠ 1 is a(n) _____ angle.

∠ 2 is a(n) _____ angle.

∠ 3 is a(n) _____ angle.

∠ 4 is a(n) _____ angle.

∠ 5 is a(n) _____ angle.

∠ 6 is a(n) _____ angle.

∠ 7 is a(n) _____ angle.

∠ 8 is a(n) _____ angle.

4. Referring to number 2, put a "T" of the following statement is true; and "F" if it is false.

__ ∠ 1 = ∠ 4

__ ∠ 1 = ∠ 5

__ ∠ 1 = ∠ 8

__ ∠ 1 = ∠ 2

__ ∠ 3 and ∠ 7 are corresponding angles

__ ∠ 4 and ∠ 3 are adjacent angles

__ ∠ 7 and ∠ 8 are alternate exterior angles

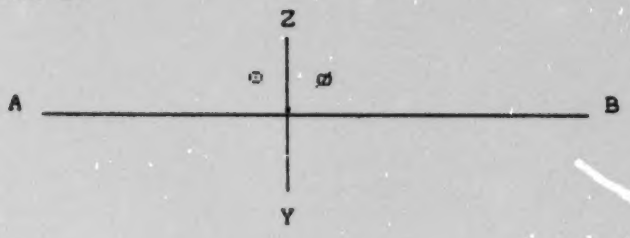
5. Name the three angles equal to ∠ 7.

STRAIGHT LINE COMBINATIONS

UNIT IV

ANSWERS TO TEST

1. and 2.



2.a. Yes. Perpendicular lines form equal, adjacent angles, each equal to 90°.

3. Exterior
Exterior
Interior
Interior
Interior
Interior
Interior
Exterior
Exterior

4. T (vertical angles)
T (corresponding angles, rule 3)
T (alternate, exterior angles, rule 2)
F (given: $\angle 1 \neq 90°$)
T
T
F (alternate exterior angles must have separate vertices)

5. $\angle 7 = \angle 6 = \angle 3 = \angle 2$

ANGULAR CONVERSION

UNIT III

UNIT OBJECTIVE

After completion of this unit, the student should be able to recall the basic definitions connected with the principal methods of angular measurement. The student should also be able to convert from one format of angular measurement to another and given two or more angles in different formats, make the appropriate conversions and determine if a given angle is larger, smaller or equal to another angle. This knowledge will be evidenced by performing the procedures outlined in the assignment sheets and by scoring 85 percent on the unit test.

SPECIFIC OBJECTIVES

After completion of this unit of instruction, the student will be able to:

1. Convert an angular measurement in fraction-of-a-degree format to Degree-Minute-Second format (DMS format)

2. Convert an angular measurement in DMS format to Decimal-Degree format (DD)

3. Convert an angular measurement in DMS format or DMS format to Fractional-Degree format.

4. Convert an angular measurement in Radians to any of the degree formats.

5. Convert an angular measurement in any of the degree formats to radians.

6. Convert an angular measurement in any of the degree formats to grads.

7. Compare angle size of two or more angles when the angles are expressed in unlike formats.

ANGULAR CONVERSION

UNIT III

SUGGESTED ACTIVITIES

I. Provide students with objective sheet.

II. Provide students with information and assignment sheets.

III. Discuss unit and specific objectives.

IV. Discuss information sheets.

A. Degree system is the most important system

(NOTE:

1. Mention that the degree-minute-second system is not a very easy system to use, but because it has been in use for thousands of years, and is the basis for the other systems, it needs to be understood.

2. The decimal-degree format solves many of the problems encountered in the DMS format.

3. The DMS format is frequently encountered in such diverse areas as rifle sights (4 clicks equal 1 minute of angle; front-end alignment specs; and navigation--a nautical mile is defined as 1' (one minute of angle) at the equator (or any other great circle about the earth.)

B. The other systems are an attempt to correct the unwieldiness of the DMS format for certain specialized applications.

(NOTE:

1. Radians is actually a measurement of angular displacement (when a wheel makes one revolution all particles of the wheel have an angular displacement of 360° or 2π radians.) As all the mechanical trades could be looked at as some branch of applied physics, the concept of angular displacement and radians will become more important as the mechanical tradesman gains competency in his field. For an angular displacement of 1 radian, the arc and the radius will be equal in length.

2. Grads at present seem to be confined mainly to the military in the field of fire control. It would also seem likely that in the future grads will also find

more applications in the civil world as well because they are a very easy way to handle angles up to 90°.

V. Pass out and discuss assignment sheets.

VI. Review unity-ratio method of conversion from one system of measurement to another.

VII. Assist students as they work through assignment sheets.

VIII. Give test.

IX. Evaluate and record test results.

INSTRUCTIONAL MATERIALS

I. Included in the unit:

A. Objective Sheet

B. Information Sheet

C. Assignment Sheets

1. Assignment Sheet #1--Convert fractional-degree format to degree-minute-second format
2. Assignment Sheet #2--Convert from DMS to DD format
3. Assignment Sheet #3--Convert from DD or DMS formats to fractional-degree format
4. Assignment Sheet #4--Convert radians to degrees
5. Assignment Sheet #5--Convert degrees to radians
6. Assignment Sheet #6--Convert degrees to grads
7. Assignment Sheet #7--Angular comparison

D. Answers to Assignment Sheets

E. Test

F. Answers to Test

References

A. Oberg, Erik and F. D. Jones. Machinery's Handbook 19th ed. New York, NY: Industrial Press, 1971.

B. Palmer, Claude I, et al. Practical Mathematics New York, NY: Gregg Division/McGraw-Hill, 1077.

C. Smith, Robert D. Vocational-Technical Mathematics Albany, NY: Delmar Publishers Incorportated, 1983.

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ANGULAR CONVERSION

UNIT III

INFORMATION SHEET

I. Terms and Definitions

A. Right Angle--1/4 of a revolution

B. Straight Angle--1/2 of a revolution

C. π (pi)--a mathematical constant

(NOTE: Common values of "pi" are: 3:14159; 3.1416; 3.14 or sometimes even 22/7 [the value used depends on the ultimate purpose or use of the answer. If the final answer needs to be accurate to the nearest 0.001" (1/1 000 th of an inch) then the 3.1416 value wold be used and the final answer rounded off to 3 decimal places].)

D. Degree--oldest method of angular measurement still in use today

1. 90° = 1/4 of a revolution

2. 180° = 1/2 of a revolution

E. Minute of Angle (')--1/60 of a degree

F. Second of Angle (")--1/60 of a minute of angle

G. Radians--π rads/2 = 1/4 revolution or 90°

1. π rads = 1/2 revolution or 180°

H. Grads--100 grads = 1/4 revolution or 90°

(NOTE: Grads are a recent attempt at simplifying angular measurement. Most dictionaries do not even mention them, but the TI-55 II calculator, for example, has a degree-radian-grad "DRG" conversion key; currently the use of grads is confined mainly to the military.)

II. Formats--With the exception of the DMS format, all of the systems are normally expressed using decimal fractions if the units do not come out exact.

Example 10.75°; 5.36 rads; 50.36 grads. In the DMS format, 50 1/2° or 50.5° would be written 50°-30'-0" (read: fifty-degrees, thirty-seconds,

zero-minutes.)

(NOTE: Although fairly common when using degrees, the common-frction format is not common in other systems.)

Example: 50 1/2° is encountered frequently but 5 1/2 rads would not be as common as 5.5 rads.

III. Methods of Conversion

A. General--Conversion from one system of angular measurement to another are made in the same way that other conversions are made.

Step 1: Discover a unity ratio with the units you want your answer to be in on the top, and the units you wish to eliminate on the bottom.

Step 2: Multiply, canceling out the units that will cancel out.

Step 3: Check your work, making sure you have not "illegally" cancelled out any units.

B. Procedures

1. Converting fractions of a degree to minutes

a. multiply the fraction (decimal or common) by 60'/degree

Example: convert 42 1/2° to Degree-Minute format

1°/2 (60'/degree) = 60'/2 = 30'

∴ 42 1/2° = 42° - 30'

Example: convert 42.50° to Degree-Minutes

0.50° (60'/degree) = .50(60') = 30'

∴ 42.50° = 42° - 30'

b. Converting Fraction of Minute-of-Angle to Second-of-Angle--Multiply the fractional portion of minute by 60"/minute

Example: Convert 42 5/16° to DMS--convert fraction of degree to minutes as in a. above

5/16° x 60'/degree = 5(60')/16 = 18 3/4'

b. Multiply fractional part of minute by 60"/1'

3/4' x 60"/1' = 3(60")/4 = 45"

∴ 42 5/16° = 42°-18'-45"

2. Convert Degrees-Minutes-Seconds to Decimal-Degree Format

   a. Convert seconds to minutes by dividing by 60 (or multiplying by 1 minute/60")

   b. Add resulting decimal fraction to minutes and divide by 60 (or multiply by 1 degree/60')

   c. Add result to whole degrees

   Example: Convert 42°-18'-45" to a decimal-degree format

   1. Divide 45" by 60:

      45/60 = 3/4 = 0.75

   2. Add to minutes:

      18' + 0.75' = 18.75'

   3. Divide minutes by 60:

      18.75'/60 = 0.3125°

   4. Add to whole degrees:

      42° + 0.3125° = 42.3125°

      (NOTE: Consulting a decimal equivalent chart, note that 0.3125 = 5/16; thus, 42.3125° = 42 5/16°)

3. Converting Decimal-Degrees to Fractional-Degree format

   a. Convert decimal fraction to common fraction by:

      1. consulting decimal equivalent chart

         or

      2. expressing as an equivalent fraction and

dividing (or reducing)

42.3125° = 42° + 3125/10 000 =

42 5/16°

4. Converting Radians to Degrees

   a. Multiply rads x 57.296°/rad

      Example: Change 1.570 rads to degrees

      1.570 rad (57.296°/rad) = 1.570(57.296°

      which equals 89.954°

   (NOTE: By definition 1 rad = 180°/π so this could be used just as easily for a conversion factor.)

5. Converting Degrees to Radians

   a. Multiply degrees x π radians/180° or degrees x 1 radian/57.296° if you prefer

      Example: Convert 270° to radians

      270°(1 radian/57.296) = 270 radians/57.296=

      4.712 radians

      or

      270°(π radians/180°) = 3π/2 radians

      or

      4.712 radians

6. Converting Degrees to Grads

   a. Multiply degrees x 100 grads/90°

      Example: Convert 45° to grads

      45°(100/grads/90°) = 45(10/9) =

      50 grads

7. To compare angles in different formats, change as many angles as required so that all angles are in the same format

Example: Which is larger?

27°45'0" or 27 5/8°

Convert the DMS format to a fraction-of-degree format and compare.

27°45' = 27 + 45/60°

reducing results in 27 3/4°, which is greater than 27 5/8°, so 27°45'0" is the larger angular measurement

ANGULAR CONVERSION

UNIT III

ASSIGNMENT SHEET #1--CONVERTING ANGULAR MEASUREMENT IN FRACTIONAL DEGREE FORMAT TO DMS FORMAT

1. What is the unity ratio that you will use to convert the common fraction part of the angular measurement to minutes-of-angle?

2. Convert the following angular measurements in fractional degree format to DMS (degree-minute-second) format:

   a. 40 1/2

   b. 35 3/4

   c. 27 5/12

   d. 55 5/6

3. What is the unity ratio to be used when the minute of angle is not exact and the fractional part of the minute of angle must be converted to second of angle?

4. Convert the following angular measurements in fractional degree format to DMS format:

   a. 29 5/16°

   b. 43 7/8°

   c. 25 3/16°

   d. 33 2/3°

5. Complete the minutes-of-angle (') and the seconds-of-angle (") columns of the chart below:

Fraction of Degree	Minute of Angle (')	Second of Angle (")
1/8		
1/4		
3/8		
1/2		
5/8		
3/4		
7/8		

ANGULAR CONVERSION

UNIT III

ASSIGNMENT SHEET #2--CONVERTING DMS FORMAT TO DD FORMAT

1. What is the unity ratio that will be used to convert the second-of-angle portion of the angular measurement to a decimal fraction expressed in minute-of-angle?

2. Eliminate the second-of-angle part of the following angular measurements by changing to a decimal fraction expressed in minutes-of-angle.

   a. 45°30"

   b. 45°45'

   c. 50°20"

   d. 58"39"

3. What is the unity ratio that will be used to convert the minute-of-angle part of an angular measurement to a decimal fraction expressed in degrees?

4. Convert the following angular measurements from DMS format to DD format by first converting the second-of-angle part of the measurement to a decimal fraction express in minutes-of-angle, adding this to the minutes-of-angle part of the measurement, and then converting this to degrees by multiplying by the unity ratio from problem 3, above.

   a. 45°30"

   b. 45°45"

   c. 50°20"

   d. 58°39"

   e. 27°30'0"

   f. 82°20'0"

   g. 56°40'15"

   h. 110°37'30"

ANGULAR CONVERSION

UNIT III

ASSIGNMENT SHEET #3--CONVERTING DD FORMAT OR DMS FORMAT TO FRACTIONAL DEGREE FORMAT

1. Convert the following angular measurements to a fractional degree format.

   a. 29.500°

   b. 53.375°

   c. 37.4514°

   d. 89.71875°

2. Convert the following angular measurements to a fractional degree format.

   a. 56°9'0"

   b. 85°18'45"

   c. 22°52'30"

   d. 59°18'45"

ANGULAR CONVERSION

UNIT III

ASSIGNMENT SHEET #4--CONVERTING RADIANS TO DEGREES

1. Using 57.296°/radian as your conversion factor, convert the following angular measurements to DD format.

   a. 10 rad

   b. 1 rad

   c. 0.8 rad

   d. 1.57 rad

2. Using 1 rad = 180°/π, calculate a rad-to-degree conversion when:

   a. π = 3.14

   b. π = 3.1416

   c. π = 3.14159

3. Using 57.296°/radian as your conversion factor, convert the following to DMS format.

   a. 0.75 rad

   b. 1.2 rad

   c. 0.875 rad

ANGULAR CONVERSION

UNIT III

ASSIGNMENT SHEET #5--CONVERTING ANGULAR MEASUREMENT FROM DEGREES TO RADIANS

1. Using either π rad/180° or 1 rad/57.296°, convert the following to radians. (assume π = 3.1416)

   a. 60°

   b. 45°

   c. 27.50°

   d. 89°45'0"

   e. 54°30'30"

   f. 35 15/16°

   g. 720°

   h. 47 7/8°

2. Would it be incorrect to express an angular measurement in terms of π rather than as a number?

ANGULAR CONVERSION

UNIT III

ASSIGNMENT SHEET #6--CONVERTING ANGULAR MEASUREMENT IN DEGREES TO GRADS

1. Using the definition for grads given in the information sheet, what is the unity ratio that will be used for converting degrees to grads?

   Reducing this, we have: grads/degrees

2. Using this ratio, convert the following to grads.

   a. 90°

   b. 45°

   c. 27°30'0"

   d. 29.375°

   e. 52°20'32"

   f. 79°45'5"

ANGULAR CONVERSION

UNIT III

ASSIGNMENT SHEET #7--ANGULAR COMPARISONS

1. Given: $\angle \alpha = 90°$

   $\angle \beta = 1.50$ rads

   $\angle \varnothing = 92$ grads

   Find: a. Which is the larger angle?

   b. Which is the smaller angle?

2. Which is larger? 52.833° or 52°53'30"

3. Which is smaller? 39.357° or 39°21'35"
   or 35.400 rads

4. Which is the larger angular measurement?

   5.1566° or 5°9'25" or 0.09 rads

ANGULAR CONVERSION

UNIT III

ANSWERS TO ASSIGNMENT SHEETS

Assignment Sheet #1

1. 60'/1°
2. a. 40°30'
   b. 35°45'
   c. 27°25'
   d. 55°50'
3. 60"/1'
4. a. 29°18'45"
   b. 43°52'30"
   c. 25°11'15"
   d. 33°40'0"
5. 7'30"
   15'0"
   22'30"
   30'0"
   37'30"
   45'0"
   52'30"

Assignment Sheet #2

1. '/60"
2. a. 45.5'
   b. 45.75'
   c. 50.33'
   d. 58.650'

3. 1'/60"

4. a. 0.7583°

   b. 0.7625°

   c. 0.83888°

   d. 0.9775°

   e. 27.5°

   f. 82.3389°

   g. 56.6709°

   h. 110.625°

Assignment Sheet #3

1. a. 29 1/2°

   b. 53 3/8°

   c. 37 2257/5000°

   d. 89 23/32°

2. a. 55 3/20°

   b. 85 5/16°

   c. 22 7/8°

   d. 59 5/16°

Assignment Sheet #4

1. a. 572.96°

   b. 57.296°

   c. 45.8368°

   d. 89.9547°

2. a. 57.3°

   b. 57.296°

   c. 57.2958°

4. a. 42°58'19.2"

   b. 68°45'18"

   c. 50°8'2.4"

Assignment Sheet #5

1. a. π/3 rads or 1.0472 rads

   b. π/4 rads or 0,7854 rads

   c. 0.480 rads

   d. 1.566 rads

   e. 0.95134 rads

   f. 0.627 rads

   g. 12.566 rads or 4 rads

   h. 0.8355 rads

2. No. Many times it would be preferable to express a measurement in terms of pi, especially if additional computations are to be done. It may that another pi will be involved and thy will cancel out; or by saving your pi until the last, your numbers will be somewhat smaller, and the likelihood of writing a wrong number down as you progress through the computations may be less. This situation is very similar to the which is better, common-fractions or decimal-fractions question. Usually decimals are better, but sometimes fractions are the best.

Assignment Sheet #6

1. 100 grads/90° or 10 grads/9°

2. a. 100 grads

   b. 50 grads

   c. 30.555 grads

   d. 32.639 grads

   e. 58.158 grads

   f. 88.613 grads

Assignment Sheet #7

1. a. angle α (alpha) at 90° is the larger angle.

   b. angle β at 92 grads (82.8°) is the smaller angle

2. a. 52°53'30" (52.833°) is the same as 52°50', which is obviously smaller than 52°53'.

3. 39.357° (39°21'25") is less that 39°21'35"

4. 5°9'25" is the larger angle

ANGULAR CONVERSION

UNIT III

TEST

1. Converting angular measurement expressed in fractional-degree format to degree-minute-second (DMS) format:

   a. What is the unity ratio used to convert the common fraction part of the angular measurement to minute-of-angle?

   b. Convert the following angular measurements in fractional-degree format to DMS format:

      1. 40 1/2°
      2. 35 3/4°
      3. 55 5/6°
      4. Give the unity ratio to convert the fractional part (if any) of minute-of-angle to second-of-angle.

   c. Convert the following angular measurements in fractional-degree format to DMS format:

      1. 1/2°
      2. 3/4°
      3. 29 5/16°
      4. 25 3/16°
      5. 33 2/3°

2. Converting from DMS format to decimal-degree (DD) format:

   a. Give the unity ratio used in converting measurements given in second-of-angle to minute-of-angle.

   b. Express the following measurements as decimal fractions with minute-of-angle as the unit of measure:

      1. 45'30"
      2. 45'45"
      3. 50'20"
      4. 58'39"

c. Give the unity ratio used to convert minute-of-angle measurement to degrees:

d. Express the following as DD format:

1. 27°30'0"

2. 56°40'5"

3. Convert the following to fractional-degree format:

a. 29.500°

b. 89.71875°

c. 56°9'0"

d. 59°18'45"

4. Convert the following to degrees (you may use any format that you wish)

a. 1 rad = 57.296°

b. 0.8 rad

c. 1.2 rad

5. Using either π rad/180° or 1 rad = 57.296°, convert the following angular measurements to radians. Assume π = 3.1416. An answer expressed in terms of pi will be acceptable.

a. 60°

b. 45°

c. 27.50°

d. 54°30'30"

e. 47 7/8°

6. Converting degrees to grads

a. What is the unity ratio for converting degrees to grads?

b. 45° is equal to _____ grads.

c. 29.375° is equal to _____ grads.

7. Which is the larger angular measurement?

5.1566°; 0.09 rads; or 5°9'25"

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ANGULAR CONVERSION

UNIT III

ANSWERS TO TEST

1. a 60'/1°

   b. 1.) 40°30'

   b. 2.) 35°45'

   b. 3.) 55°50'

   b. 4.) 60"/1'

   c. 1.) 30'0"

   c. 2.) 45'0"

   c. 3.) 29'18'45"

   c. 4.) 25'11'15"

   c. 5.) 33'40'0"

2. a. 1'/60"

   b. 1.) 45.5'

   b. 2.) 45.75'

   b. 3.) 50.333'

   b. 4.) 58.650'

   c. 1°/60'

   d.) 1. 27.50°

   d.) 2. 56.6709°

3. a. 29 1/2°

   b. 89 23/32°

   c. 56 3/20°

   d. 59 5/16°

4. a. (example -- not to be counted)

   b. 45.8368°

   c. 68°45'18"

5. a. π/3 rads or 1.0472 rads

   b. π/4 rads or 0.7854 rads

   c. 0.4799 rads or 0.480 rads

   d. 0.9513 rads

   e. 0.8355 rads

6. a. 100 grads/90° or 10 grads/9°

   b. 50 grads

   c. 32.639 grads

7. 5°9'25" is the larger measurement (its decimal equivalent is 5.1569°, whereas 0.09 rads is equal to 5.15664°)

CIRCLES

UNIT IV

UNIT OBJECTIVE

After completion of this unit, the student should be able to recall certain terms and definitions related to circles. The student should also be able to recall additional terms and relationships that exist when circles are intersected by straight lines. This knowledge will be evidenced by performing the procedures outlined in the assignment sheets and by scoring 85 percent on the unit test.

SPECIFIC OBJECTIVES

After completion of this unit of instruction, the student will be able to:

1. Given a drawing of the circle intersected by certain combinations of lines and line segments, and angles.

   a. label and identify the arc, center, central angle, diameter, chords, radii and circular segment.

   b. distinguish between chords and radii

   c. identify the relationship between the radii and the diameter.

2. Given a list of certain terms related to the circle, match these terms with appropriate definitions.

CIRCLES

UNIT IV

SUGGESTED ACTIVITIES

I. Make transparency.

II. Provide students with objective sheet.

III. Provide students with information and assignment sheets.

IV. Discuss unit and specific objectives.

V. Discuss information and assignment sheets.

A. Explain original meaning of Latin and Greek root words

1. tangent--touching; "tangible assets" means assets that can be physically touched, thus the tangent line is the line that touches the circle

2. secant--from Latin, secare, to cut. Our word "sickle", to describe a tool used for cutting grass comes from this word; a secant line is the line that cuts through the circle

B. Review applications relevant to class potential needs

C. Review use of compass if required

VI. Assist students as they work through assignment sheets.

VII. Give test.

VIII. Evaluate and record test results.

INSTRUCTIONAL MATERIALS

I. Included in the unit:

A. Objective Sheet

B. Information Sheet

C. Transparency Master

D. Assignment Sheets

1. Assignment Sheet #1--Labeling parts of circle; relationship of radii to diameter

2. Assignment Sheet #2--Drawing and labeling circles, center, tangent, diameter and radius

3. Assignment Sheet #3--Matching terms with definitions

E. Answers to Assignment Sheets

F. Test

G. Answers to Test

II. References

A. Oberg, Erik and F.D. Jones. Machinery's Handbook, 19th ed.. New York, NY: Industrial Press, Inc., 1971.

B. Smith, Robert D. Vocational-Technical Mathematics Albany, NY: Delmar Publishers Incorporated, 1983.

CIRCLES

UNIT IV

INFORMATION SHEET

I. Terms and Definitions

A. Circle--a closed curve, all points of which are equidistant from a fixed point within called center

B. Center--the fixed point enclosed by a circle which is equidistant from all points on the circle

C. Circumference--the length of the curved line which forms the circle

D. Radius--the distance from the center of a circle to the curved line; a straight line segment which connects the center to any point on the curved line

(NOTE: All circles have a radius, whether or not it is actually drawn in as a line or not.)

E. Arc--a segment of the curved line forming a circle

(NOTE: Because arcs are curved line segments, they must have end points. An arc bounded by points "A" and "B" would be written " AB " and read as "arc AB".)

F. Relationships between a circle and a straight line

Three possible relationships:

1. The line does not touch the circle at all

(NOTE: If this is the case, the possibility needs no further discussion.)

2. The line intersects the circle at two points

(NOTE: Remember, we are talking about straight lines which are infinitely long , as opposed to line segments which have a specific length. Also note that it would not be possible for a straight line to intersect a circle at more than two points.)

A. A line which intersects a circle in two points is called a "secant"

B. The segment of the secant that lies within the circle is called a "chord". A chord, then, is a line segment connecting two points on a circle

C. A chord passing through the center of a circle is called the "diameter"

3. The last possibility is that the line touches the circle at one point and one point only

A. A line which touches a circle at one point and one point only is called a "tangent"

B. The point where the line touches the circle is called the point of tangency

4. Areas formed by chords and radii

A. The area of a circle enclosed by two radii is called a "sector"; if a round pizza is cut in the usual way, each piece is actually a sector

B. The area enclosed by a chord and its arc is called a "circular segment" or if it is understood to be circular, simply "segment"

G. Central angle--the angle formed by two radii and which has the center as its vertex

1. The size of the central angle is directly related to arc length, chord length, sector area and segmental area

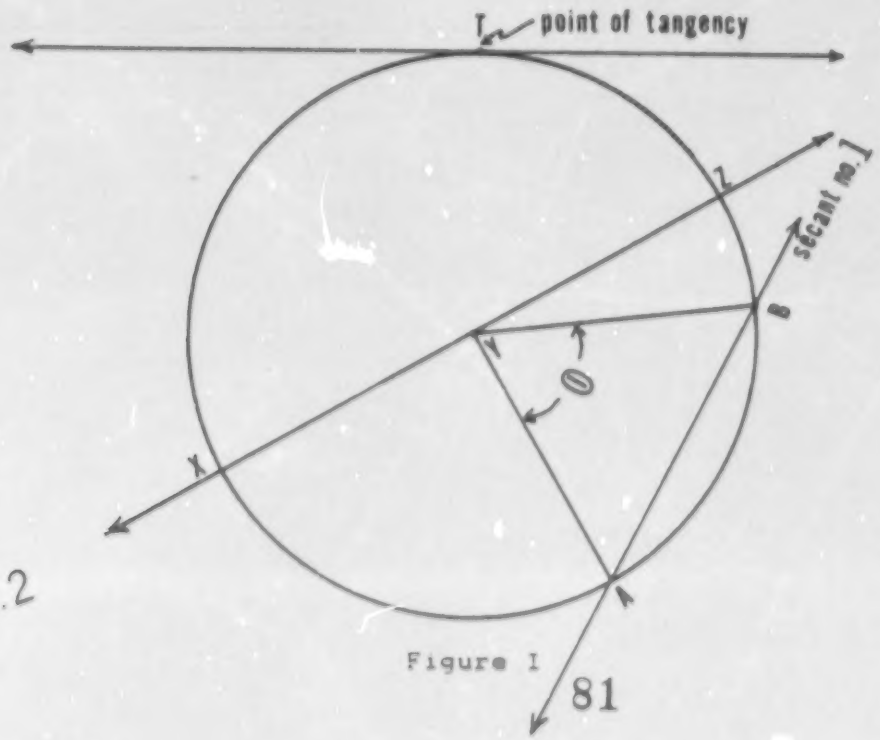


Figure I

H. Figure I shows the relationships of the various terms and definitions just discussed.

1. The line marked "tangent" is a tangent because it touches the circle at one point and one point only, point T which is the point of tangency

2. The center of the circle is designated by the letter "Y".; the circle referred to in Figure I could be referred to as "circle Y"

3. Secant #1 is a secant because it intersects the circle at two points, point A and point B. Line segment AB is a chord, which also defines arc AB as well; the enclosed area is called segment AB

4. Secant #2 not only intersects the circle at points X and Z, but also passes through the center, Y, as well; line segment XZ is not only a chord but a diameter as well

5. Line segments YA and YB are radii; angle AYB has been labeled θ (theta) and is the central angle

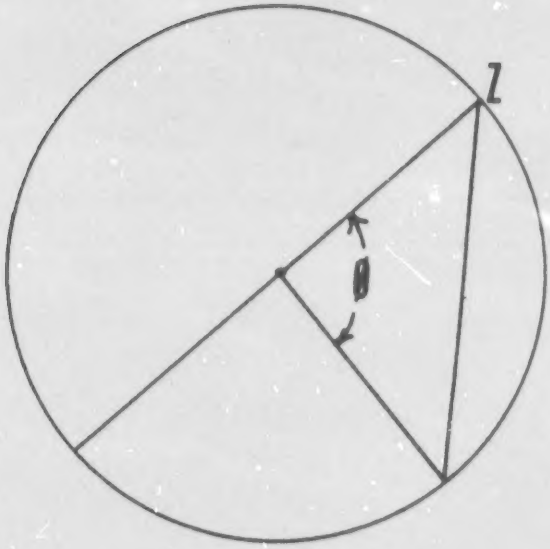
6. The area of triangle YAB and the area of segment AB together make up the circular sector defined by central angle theta

7. The circumference would be the distance around the circle, or more correctly, the length of the curved line forming the circle; the distance from point T all the way around, passing points Z, B, A, X, and back to point T again would be equal to the circumference

CIRCLES

UNIT IV

ASSIGNMENT SHEET #1--LABELING PARTS OF CIRCLE; RELATIONSHIP OF RADII TO DIAMETER



1. Label the center of the circle "O".
2. Label the diameter XZ.
3. Label the chord which is not a diameter ZG.
4. Identify the two central angles using the three letter format. (Remember: the vertex must be the middle letter.)
5. Shade the circular segment that is defined by angle ø.
6. Does $\overline{XO} = \overline{OG}$? Why?
7. Does $\overline{XO} = \overline{OX}$? Why?
8. Is $\overline{GZ}$ a radius? Why?

CIRCLES

UNIT IV

ASSIGNMENT SHEET #2- DRAWING AND LABELING CIRCLES, CENTERS, TANGENT, DIAMETER AND RADIUS

1. Using a compass, draw a circle with a radius equal to 2".

   a. Label the center C

   b. Using a straight edge, draw a tangent to the circle

   c. Label the point of tangency X

   d. Draw a diameter, and label the points of intersection with the circle, points A and B

   e. Draw a radius, CG

CIRCLES

UNIT IV

ASSIGNMENT SHEET #3--MATCHING TERMS WITH DEFINITIONS

At the left is a list of terms related to the circle. On the right are definitions that may or may not correspond to these terms. Match the definitions with the term by placing the identifying letter in the blank to the right of the appropriate term.

___ 1. Center

___ 2. Tangent

___ 3. Radius

___ 4. Segment

___ 5. Secant

___ 6. Arc

___ 7. Chord

___ 8. Central Angle

___ 9. Circle

a. a geometric figure created by a closed curved line, all points of which are an equal distance from a fixed point inside the figure

b. a straight line which intersects the circle in two points

c. a line segment connecting two points on a circle

d. the distance around the circle

e. the angle formed by two radii and which has the center as its vertex

f. the fixed point within the circle which is equidistant from all points on the curved boundry line

g. a straight line which touches the circle at one point only

h. a line segment that connects the center of a circle with any point on the circle

i. a part of the circle bounded by two points; a circular line segment

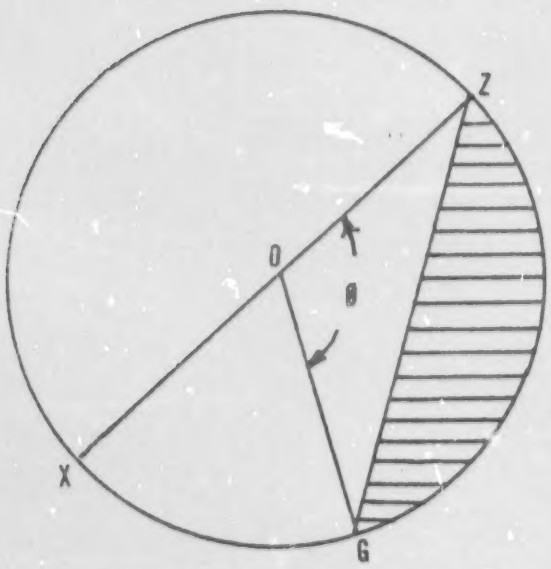
j. the area of a circle contained by an arc and the chord of the arc

CIRCLES

UNIT IV

ANSWERS TO ASSIGNMENT SHEETS

Assignment Sheet #1



1., 2., 3. Evaluated to the satisfaction of the instructor.

4. ∠ XOG and ∠ GOZ

5. Evaluated to the satisfaction of the instructor.

6. Yes. They are both radii and radii of the same circle are equal.

7. Yes. They are both radii and radii of the same circle are equal.

8. No. It does not connect the center with a point on the circle.

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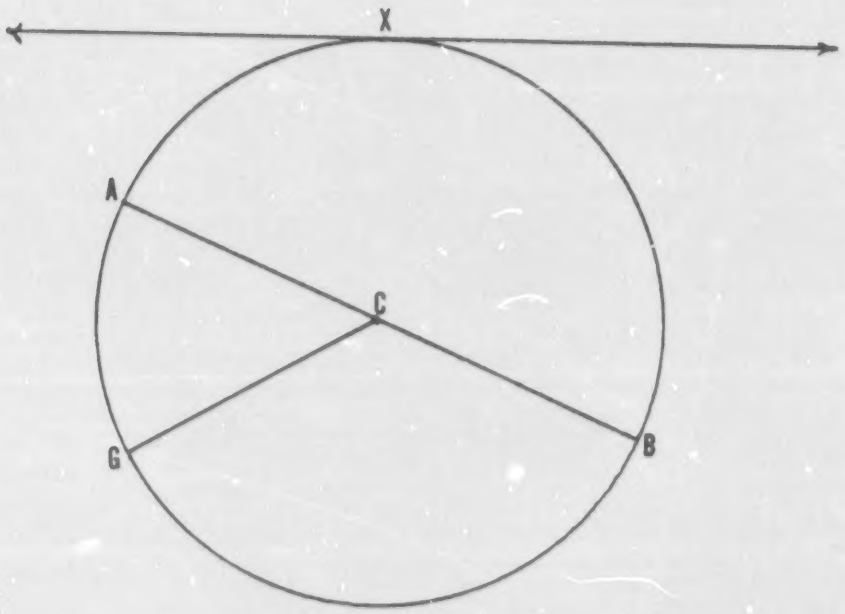
Assignment Sheet #2



Evaluated to the satisfaction of the instructor.

Criteria:

Lines may be at any angle.
The diameter must pass through the center, C.
Radius CG may be at any angle. G must be on the circle

Assignment Sheet #3

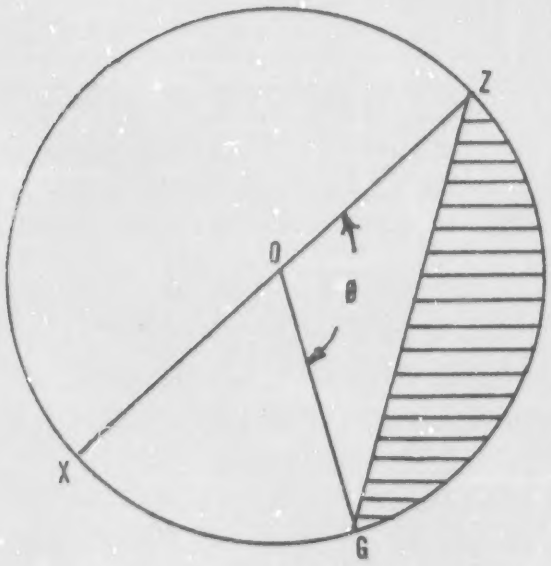
1. f
2. g
3. h
4. j
5. b
6. i
7. c
8. e
9. a

CIRCLES

UNIT IV

ANSWERS TO ASSIGNMENT SHEETS

Assignment Sheet #1



1., 2., 3. Evaluated to the satisfaction of the instructor.

4. ∠ XOG and ∠ GOZ

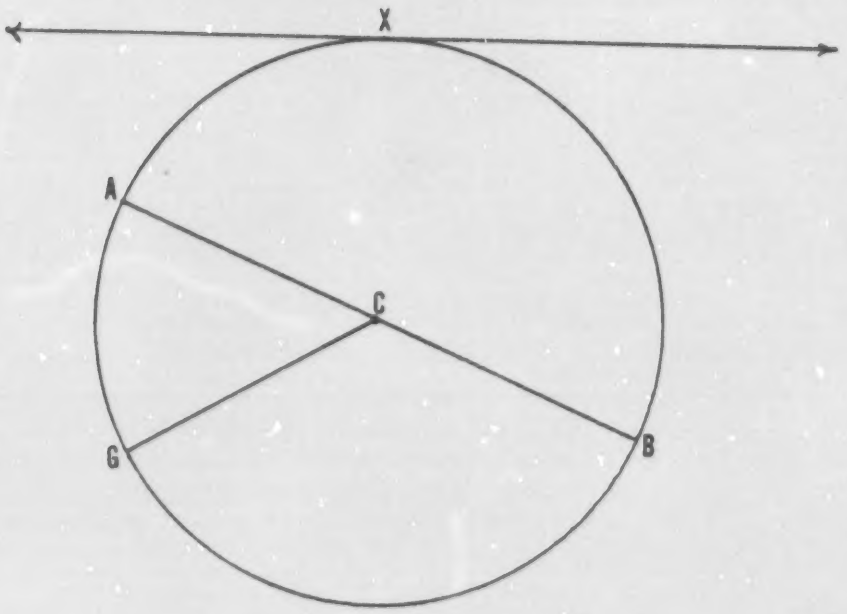
5. Evaluated to the satisfaction of the instructor.

6. Yes. They are both radii and radii of the same circle are equal.

7. Yes. They are both radii and radii of the same circle are equal.

8. No. It does not connect the center with a point on the circle.

Assignment Sheet #2



Evaluated to the satisfaction of the instructor.

Criteria:

Lines may be at any angle.
The diameter must pass through the center, C.
Radius CG may be at any angle. G must be on the circle

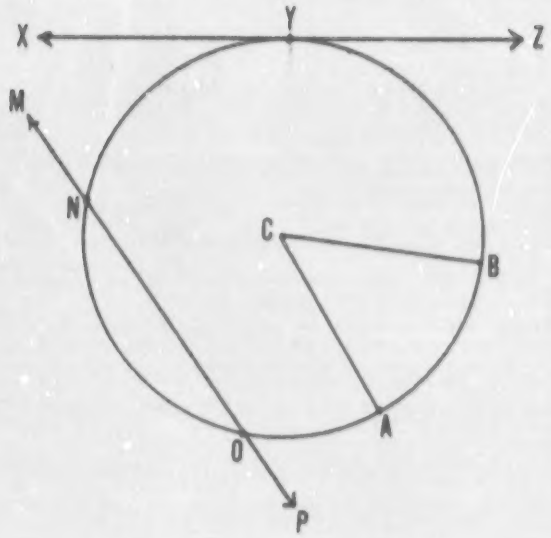
Assignment Sheet #3

1. f
2. g
3. h
4. j
5. b
6. i
7. c
8. e
9. a

CIRCLES

UNIT IV

TEST



1. and 2.

Answer the following by writing the correct answer in the blank.

a. Which line is a tangent? ____
b. Which point is the point of tangency? ____
c. Which line is a secant? ____
d. Which line segment is a chord? ____
e. The center of the circle is point? ____
f. Name the central angle that corresponds to arc AB.___
g. Shade in the circular segment.
h. Write the word "SECTOR" inside the sector of the circle.

3. Match the terms on the left with the definition on the right by writing in the correct letter in the blank to next to the term.

___ 1. Center
___ 2. Central Angle
___ 3. Radius
___ 4. Circle

a. a geometric figure created by a closed curved line, all points of which are an equal distance from a fixed point inside the figure

b. a line segment that connects the center of a circle with any point on the circle

c. the fixed point within a circle that is equidistant from all points on the circle

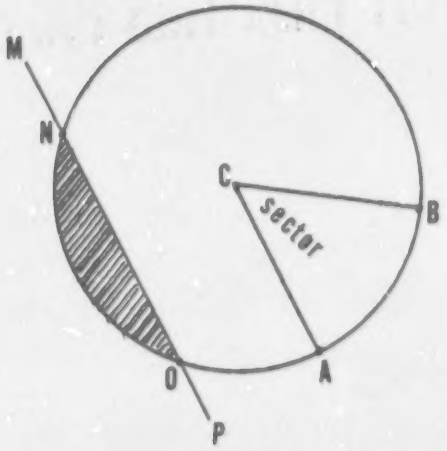
d. the angle formed by two radii, with the center of the circle as its vertex

e. a part of the circle bounded by two points; a circular line segment

CIRCLES

UNIT IV

ANSWERS TO TEST



1. and 2.

a. XZ
b. Y
c. MP
d. NO
e. C
f. Angle ACB
g. Shaded to the satisfaction of the instructor

3. 1. c
   2. d
   3. b
   4. a

POLYGONS

UNIT V

UNIT OBJECTIVE

After completion of this unit, the student should be able to identify basic polygons visually and by name. The student should be able to distinguish between inscribed polygons and circumscribed polygons. The student should also be able to make general classifications of polygons and apply this knowledge in the solution of certain practical problems. This knowledge will be evidenced correctly performing the procedures outlined in the assignment sheets and by scoring 85 percent on the unit test.

SPECIFIC OBJECTIVES

After completion of this unit of instruction, the student will be able to:

1. Visually identify and/or draw concave and convex polygons with up to nine sides.
2. Classify quadrilaterals as being: trapezoids, trapeziums, parallelograms or rectangular parallelograms.
3. Classify polygons as being regular or irregular.
4. Identify internal and external angles of polygons and using the appropriate formula, determine their measure if the polygon is regular.
5. Distinguish between circumscribed and inscribed polygons.
6. Apply certain key constant ratios concerning inscribed/circumscribed polygons and their circles in the solution of practical problems.

POLYGONS

UNIT V

SUGGESTED ACTIVITIES

I. Make transparencies.

II. Provide students with objective sheet.

III. Provide students with information and assignment sheets.

IV. Discuss unit and specific objectives.

V. Discuss information and assignment sheets.

VI. Illustrate concepts with appropriate transparencies.

VII. Assist students as they work through assignment sheets.

VIII. Give test.

IX. Evaluate and record test results.

X. Review corrected tests with students.

INSTRUCTIONAL MATERIALS

I. Included in the unit:

A. Objective Sheet

B. Information Sheet

C. Transparency Masters

1. TM#1--Exterior angles

2. TM#2--Inscribed and circumscribed polygons

3. TM#3--Constant ratios

D. Assignment Sheets

1. Assignment Sheet #1--Draw and identify polygons

2. Assignment Sheet #2--Identify and name quadrilaterals

3. Assignment Sheet #3--Identify and calculate interior and exterior angles

4. Assignment Sheet #4--Practical applications of concepts

E. Answers to Assignment Sheets

F. Test

G. Answers to Test

II. References

A. Levine, Samuel. Vocational and Technical Mathematics in Action. Rochelle Park, NJ: Hayden Book Company, Inc., 1978.

B. Smith, Robert D. Vocational-Technical Mathematics Albany, NY: Delmar Publishers Incorporated, 1983.

POLYGONS

UNIT V

INFORMATION SHEET

I. Terms and Definitions

A. Polygons (General)

1. The word "polygon" comes from two Greek words--"poly" meaning many and the word "-gon" meaning side

(NOTE: Think of terms like poly- phase current as opposed to single-phase current; a polytechnical school which teaches several technical subjects.)

2. A polygon is a geometric figure with many sides; the actual definion is: " a closed, plane figure made up of three or more straight line segments"

(NOTE: Could you draw a closed figure with only 2 sides?)

3. The Greeks and Romans loved to classify things and many of their names for polygons are in use today.

a. Triangle (three angles) describes a three-sided polygon

(NOTE: Actually, the Greeks called this a "trigon" or "threesides"and the Romans decided to call it a "three-angle"; trigonometry still uses the Greek word to describe the measurement of "trigons".)

b. Quadrilaterals--have four sides

(NOTE: Four-barreled carburetors are sometimes called "quads"; a "lateral" in football is to throw off to one side.)

c. Pentagons--have five sides; "pent" means five

d. Hexagons--have six sides

e. Octagon--have eight sides

(NOTE: An octopus has eight legs; there are eight notes in an octave.)

f. Nonogon--has nine sides

g. Decagons--have ten sides

(NOTE: Our word decimal comes from the Greek work "dekos" for ten.)

4. All polygons must be closed figures meaning that all sides must be connected.

5. There are five basic kinds of polygons

a. concave polygons--polygons that if two or more sides are extended, one or more of the extended sides will cut through the polygon

b. convex polygon--one in which no side will cut into the polygon no matter how much the sides are extended

c. equilateral polygons--where all sides are the same length

d. equiangular convex polygons--where all angles are equal

e. regular polygons--convex polygons that are also equilateral and equiangular

II. Special Cases and Subdivisions of Polygons

A. Triangles

1. Triangles are often further classified according to properties of their sides

a. equilateral triangles--all sides are equal

(NOTE: They also happen to be equiangular, but this coincidence does not continue with other polygons.)

b. scalene triangles--have all sides unequal

c. isosceles triangles have two sides equal

2. Triangles are also classified according to properties of their angles

a. equiangular triangles have all their angles equal

(NOTE: They also happen to be equilateral as mentioned above.)

b. acute triangles--where all angles are less than 90°

c. obtuse triangles--where one angle is greater than 90°

d. right triangles--where one angle is equal to 90°

B. Quadrilaterals

1. Trapeziums--four-sided figures where no sides are equal and no sides are parallel

2. Trapezoids--four-sides figures where two sides are parallel

3. Parallogram--four-sided figure with opposite sides parallel and equal

(NOTE: This does not mean that ALL four sides are equal.)

a. a rectangle is a parallelogram which is also equiangular which means that all four angles are equal to 90°; an equilateral rectangle is also called a square; squares are the only quadrilaterals that are regular polygons

b. a rhombus is a parallelogram with all four sides equal, but which is NOT equiangular

(NOTE: The rest of the polygons are simply classified as in Section I-5, that is: concave, convex, equianglar, equilateral, or regular.)

III. Special Characteristics of Polygons

A. Angles

1. Interior angles of polygons--the angles formed by sides of the polygon with each corner being the vertex; the number of interior angles is equal to the number of sides

2. Exterior angles--the angles formed by extending a side, and are outside the polygon

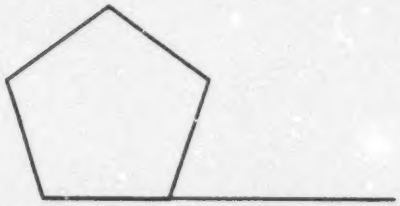


Figure III:A:2

(NOTE: Polygon ABCDE is a regular pentagon. Side AB has been extended to point X. ∠ CBX is an exterior angle. ∠ ABC is an interior angle. ∠s ABC and CBX are supplementary.)

3. The sum of the interior angles of a polygon is equal to (N-2)(180°), where N = number of sides.

   Example: What is the sum of the interior angles of a quadrilateral?

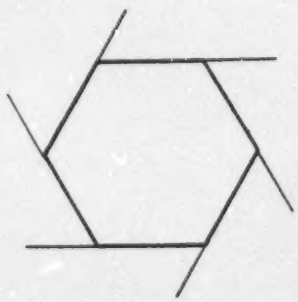
   Solution: a quadrilateral has 4 sides

   ∴ N = 4 and N-2 = 4-2 or 2

   so sum of interior angles of a quadrilateral is equal to

   2 (180°) or 360°

4. The sum of the exterior angles of a polygon, if each side is extended in succession, is 360°



Hexagon with each side extended in succession and exterior angle numbered 1-6

Figure III:A:4

5. For regular polygons, because all angles are equal, any interior angle is equal to:

$$\frac{(N-2)(180°)}{N}$$

and any exterior angle is equal to

$$\frac{360°)}{N}$$

Example: What is the angular measurement of an interior angle of a square?

Solution:

a square has four sides so N = 4

substituting:

$$\text{int. } \angle = \frac{(N-2)(180°)}{N} =$$

$$\frac{(4-2)(180°)}{4} =$$

$$\frac{2(180°)}{4} =$$

$$\frac{360°)}{4} = 90°$$

Each interior angle of a regular quadrilateral = 90°

(NOTE: The sum of interior angles for polygons 5 or more sides will be greater than 360°.)

IV. Inscribed and Circumscribed Polygons

A. a polygon is inscribed in a circle if all vertices (corners) of the polygon lay on the circle

B. A polygon is circumscribed about a circle if all vertices are outside the circle and the sides are tangent to the circle

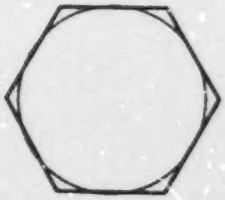


Fig. I

Fig. II

Figure I shows a hexagon circumscribed about a circle. If this is a regular hexagon, the points of tangency will bisect each side.

Figure II shows hexagon inscribed in a circle. Notice that each vertex is on the circle and that each side is a chord. A line connecting two non-adjacent vertices is called a diagonal. AB is a diagonal . Diagonals connecting opposite sides of regular polygons with an equal number of sides are equal to the diameter of the circle.

Fig. III

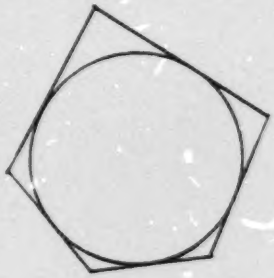


Fig. IV

Figures III and IV show that irregular polygons can be inscribed and circumscribed as well.

C. Ratios of circumscribed and inscribed regular polygons

1. There are certain constant ratios that are true for specific polygons, regardless of size. In the following:

   a. "D" is the diameter of the circumscribed circle
   b. "d" is the diameter of the inscribed circle
   c. "a" is the side of the polygon
   d. "h" is the altitude of heights of the polygon (the perpendicular distance from one side to the point opposite--which may be another side in an even-sided polygon, on a vertex is an odd-sided polygon)
   e. "A" is the area enclosed by the polygon

2. In a regular triangle (an equilateral triangle is the only triangle that satisfied the requirements for a regular polygon), the ratios are:

$$\frac{d}{D} = 0.500 \qquad \frac{A}{a^2} = 0.433$$

$$\frac{a}{D} = 0.866 \qquad \frac{h}{a} = 0.866$$

$$\frac{h}{D} = 0.750$$

3. For squares (regular quadrilaterals) the ratios are:

$$\frac{d}{D} = 0.707 \quad \text{also became } a = d \quad \text{then } \frac{a}{D} = 0.707$$

$$\frac{A}{d^2} = 1.000$$

$$\frac{A}{D^2} = 0.500$$

a. The most important of these ratios is

$$\frac{d}{D} = 0.707 \text{ or } \frac{a}{D} = 0.707$$

b. The second ratio is so obvious, it need not be remembered.

c. The third ratio has little practical value and need not be remembered either.

(NOTE: The last two ratios are mentioned only to show that constant ratios can be derived for all regular polygons, but that not all knowledge is necessarily useful.)

3. For regular pentagons, the ratios are:

$$\frac{d}{D} = 0.809$$

$$\frac{a}{D} = 0.588$$

$$\frac{h}{D} = 0.904$$

$$\frac{A}{a^2} = 1.720$$

4. For hexagons the ratios are:

$$\frac{d}{D} = 0.866 \quad \text{because } d = h \quad \frac{h}{D} = 0.866$$

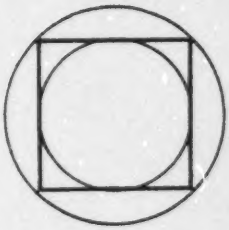
$$\frac{a}{D} = 0.500$$

$$\frac{A}{a^2} = 2.598$$

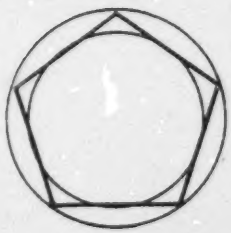
D. Illustrations

Constant Ratios

D is the diameter of the circumscribed circle; d is the diameter of the inscribed circle; a is the length of the side of the polygon; h is the altitude of the polygon



(NOTE: For a square, a = d = h and D is also equal to the diagonal.)



(NOTE: For a hexagon, d = h and h is the distance across the flats. D is equal to the diagonal.)

POLYGONS

UNIT V

ASSIGNMENT SHEET #1--DRAW AND IDENTIFY POLYGONS

1. Using a straight edge, draw:

   a. a concave quadrilateral

   b. a convex quadrilateral

   c. a convex hexagon

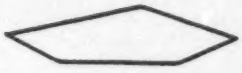
   d. a convex octagon

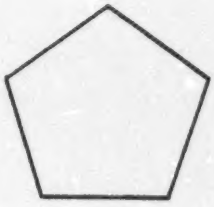
2. Write the name of the following polygons in the blank to the right of the polygon. Choose from the list of possibilities at the far right.

a. _____

b. _____

c. _____

d.  _____

e.  _____

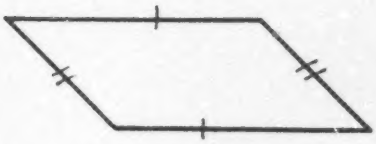
Quadrilateral
Hexagon
Octagon
Nonogon
Pentagon
Triangle

POLYGONS

UNIT V

ASSIGNMENT SHEET #2--IDENTIFY AND NAME QUADRILATERALS

1.



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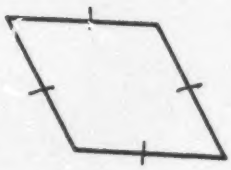

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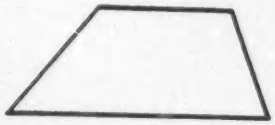

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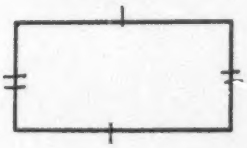

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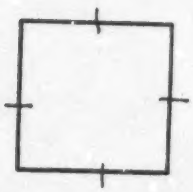

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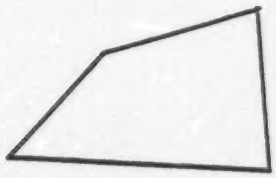


_____

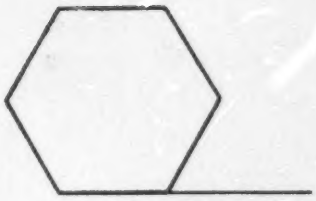
Identify the above quadrilaterals using the specific name for each. (Equal sides have the same number of "tick-marks". Assume sids are parallel unless this is clearly not the case.)

2. Which of the above are regular?

POLYGONS

UNIT V

ASSIGNMENT SHEET #3--IDENTIFY AND CALCULATE INTERIOR AND EXTERIOR ANGLES



Polygon ABCDEF is a regular hexagon, EX is an extension of side ED.

1. What is the sum of interior angles?

2. What is the angular measurement of angle ABC?

3. What is the angular measurement of angle AFE?

4. What is the angular measurement of angle CDX:

   a. Using the fact that ∠ EDC and ∠ CDX are supplementary, what is the angular measurement of ∠ CDX?

   b. Using the formular method, calculate the measure of ∠ CDX.

POLYGONS

UNIT V

ASSIGNMENT SHEET #4--PRACTICAL APPLICATIONS OF CONCEPTS

1. A machinist is to layout a 3-hole bolt pattern. The drawing says 1/2" Dia, thru 3 places, equidistant on a 5" B.C. (NOTE: B. C. stands for bolt circle.) What distance will he set his dividers at for the preliminary layout?

2. A contractor is setting the "J-bolts" for a 4-legged antenna tower, which he does not have access to. The manufacturer's drawing shows the j-bolt holes to be equidistant on a 24" diameter bolt circle. What is the center distance between bolts? What should the diagonal measurement be?

3. What is the diameter of the smallest log that could be used to yield a 10" x 10" railroad tie?

4. Geodesic domes often have a pentagonal foundation. If the pentagonal foundation for a small half-dome is to have 5 ft. to a side, what is the diameter of the circle that the builder will layout to find the location of the outside corner of the foundation?

5. What is the smallest diameter of round bar that will yield a hexagon with a 2" dimension across the flats? If the hex is to be cut on a vertical milling machine, what is the depth of cut?

6. To layout an equidistant 6-hole bolt pattern, an old "rule of thumb" says to space the holes equal to the radius of the bolt circle. Is this consistent with our constant ratios? Why?

POLYGONS

UNIT V

ANSWERS TO ASSIGMENT SHEETS

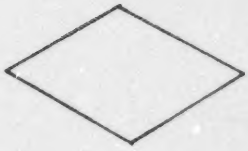
Assignment Sheet #1

1.

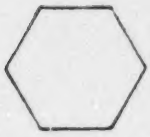
a.

Criteria:

must be closed;
must have four sides (or lines);
two sides must "point" into the polygon

b. 

must be closed;
must have 4 sides;
all corners must point out;
sides need not be equal

c. 

per above except there must be six sides

d.

per above except there must be eight sides

2. a. triangle
   b. quadrilateral
   c. hexagon
   d. pentagon
   e. pentagon

3. "b" is also a parallelogram

4. "e" appears to be a regular polygon--all sides look equal as do the angles

5. No. It is not a closed figure.

Assignment Sheet #2

1. a. parallelogram
   b. rhombus
   c. trapezoid
   d. rectangle
   e. square
   f. quadrilateral (No other specific name because it clearly is not a parallelogram)

2. "e" only squares satisfy all the conditions for being regular as they are not only equilateral, but equiangular as well.

Assignment Sheet #3

1. sum int. ∠s = (N-2)(180°) Hexagon N = 6
   = (4)(180°) N-2 = 4
   = 720°

2. ∠ ABC is an interior ∠
   all interior ∠ s are equal in a regular polygon
   ∠ ABC = 720°/6 = 120°

3. ∠ AFE = ∠ ABC = 120°

4. Using the supplementary property, we have already determined that each interior ∠ = 120°

   ∠ EDC is an interior ∠

   ∠ EDC + ∠ CDX = 180°

   120° + ∠ CDX = 180°

   ∠ CDX = 180° - 120°

   ∠ CDX = 60°

   Using formula method:

   sum exterior angles = 360°
   for a regular hexagon, a given exterior angle = 360°/6 or 60°

   ∠ CDX is an exterior ∠, ∴ ∠ CDX = 60°

Assignment Sheet #4

1. D = 5" a/D = 0.866

   (a/5 = 0.866)5

   a = 5(0.866); a = 4.33"

2. D = 24" a/D = 0.707

a/24" = 0.707

a = (0.707)24

a = 16.960" (read on his tape as 16 31/32")

The diagonal measurement is the same as the diameter, 24".

3. a = 10"

a/D = 0.707

a/0.707 = D

10/0.707 = D

14.14" = D, minimum log diameter

4. a/D = 0.588

5'/D = 0.588

5'/0.588 = D

8.503' = D

5. h = 2" h/D = 0.866

2"/D = 0.866

2"/.0866 = D

2.309" = D or min diameter

$$\text{depth of cut} = \frac{D-d}{2} \quad \text{or} \quad \frac{2.309 - 2}{2} = \frac{.309}{2} = 0.1545$$

6. Yes. Our ratio says a/D = 0.500 or a/D = 1/2
2a = D; a = D/2, which is the same as the radius

POLYGONS

UNIT V

TEST

1. Using a straight edge, draw the following:

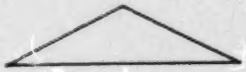
   a. convex quadrilateral

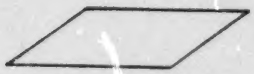
   b. concave quadrilateral

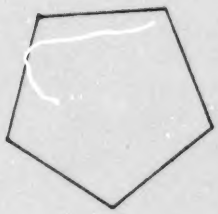
   c. convex hexagon

   d. convex octagon

e. Write the name of the following polygons in the blank to the right of each figure. Choose from the list of possibilities on the far right.

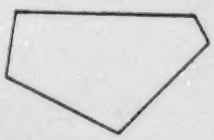
1.  _____

2.  _____

3.  _____

4. _____

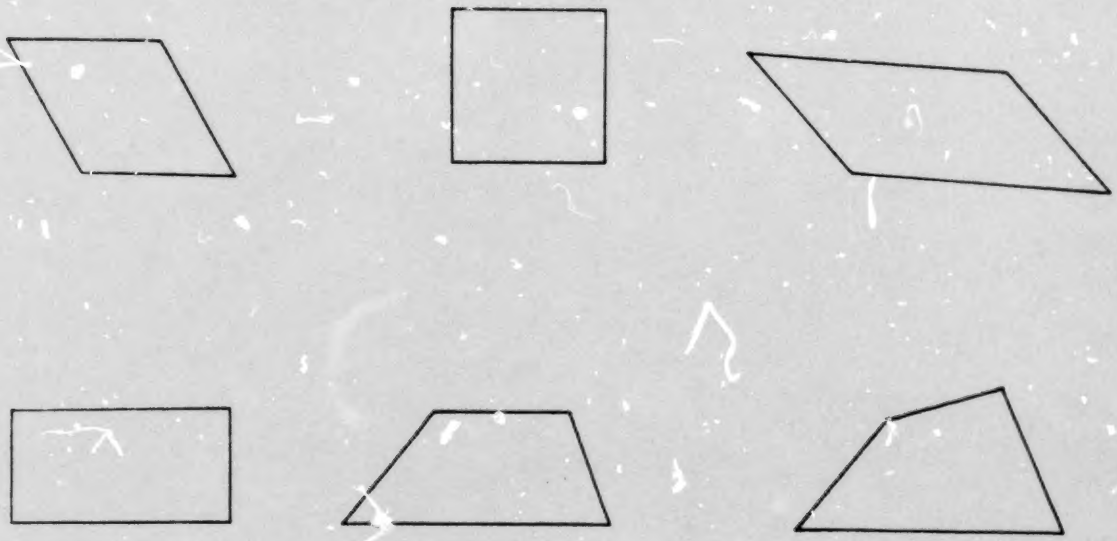
5. _____



quadrilateral
hexagon
octagon
nonogon
pentagon
triangle

f. Which of the polygons in "e" is likely to be a parallelogram?

2.



Identify the above quadrilaterals using the specific name for each. (Equal sides have the same number of "tick-marks".)

a. _____ d. _____
b. _____ e. _____
c. _____ f. _____

3. Which of the above satisfies all the requirements of a regular polygon?

4. a. Sketch a nonogon.

b. Extend a side and name the exterior angle ∠ :
Name the interior angle ∠ ;

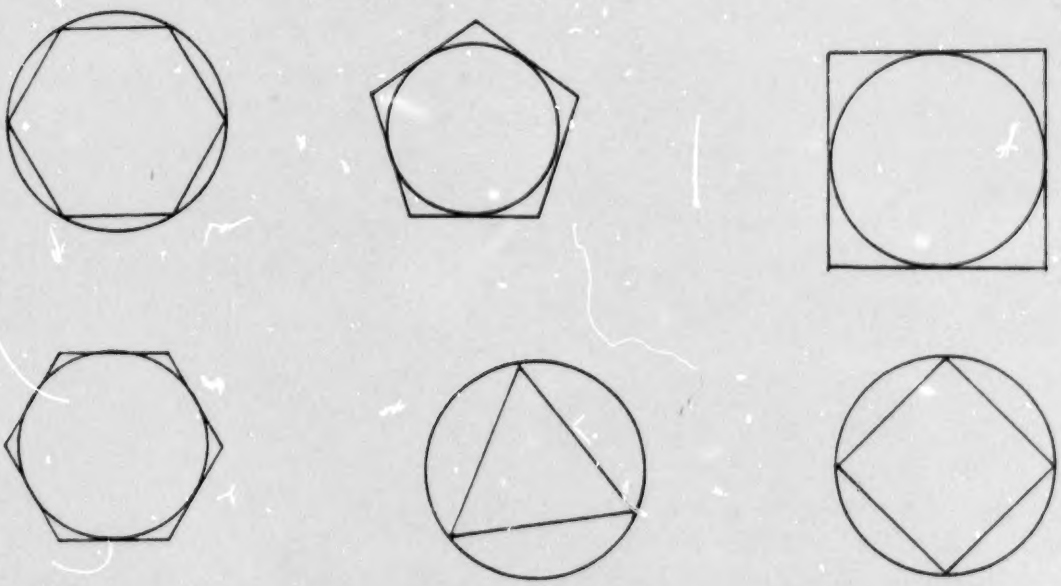
c. If your polygon is regular, what is angular measurement of ∠ c in degrees?

d. If your polygon is regular, what is angular measurement of ∠ e in degrees?

e. Assuming your polygon is regular, solve for measurement of ∠ e using a method other than the one you used 'd'.

5. a. Which of the following are inscribed polygons?

b. Which are circumscribed?



6. a. What is the largest size square that can be machined from a 1" diameter piece of stock?

b. What is the largest size hex that can be machined from a 1" diameter piece of stock?

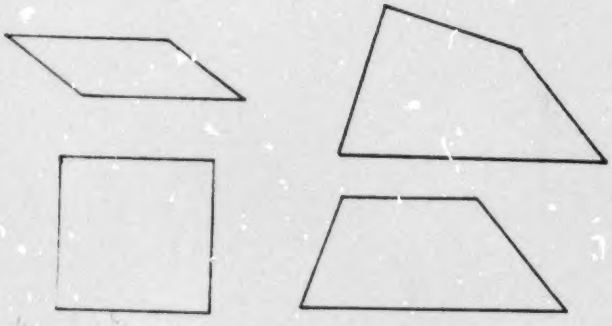
c. What is the center distance between holes of a 6 pattern equidistant bolt pattern on a 12" bolt circle?

POLYGONS

UNIT V

ANSWERS TO TEST

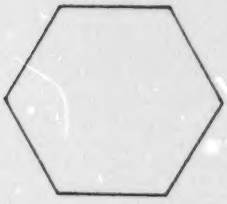
1. a.



anything similar to the above is acceptable; must be 4 lines, all connected; all corners must point out

b.

c.



(sides need not be equal; angles need not be equal; must be six sides; all corners pointing out)

d.

(per above except there must be eight sides)

e. 1. triangle
2. quadrilateral
3. pentagon
4. hexagon
5. pentagon

f. e.2. only quadrilaterals can be parallelograms

2. a. rhombus
b. square
c. parallelogram
d. rectangle
e. trapezoid
f. trapezium

3. b - square

4. a and b

c. 360/9 = 40°

d. 180° - 40° = 140°

e. $\frac{(N-2)(180)}{N} = \frac{7(180)}{9} = 7(20°) = 140°$

5. a. a; e; f
b. b; c; d

6. a. d/D = 0.707

d/1 = 0.707

d = 0.707 so a is a 0.707" x 0.707" square

b. d/D = 0.866

d/1 = 0.866

c. 12/2 = radius and center distance

6" = radius on center distance

GEOMETRIC FORMULAS

UNIT VI

UNIT OBJECTIVE

After completion of this unit, the student should be able to use standard geometric formulas with accuracy. This knowledge will be evidenced by correctly performing the procedures on the assignment sheets and by scoring 85% on the unit test.

SPECIFIC OBJECTIVES

After completion of this unit of instruction, the student will be able to:

1. Compute sides of right triangle using Pythagorean Theorem.
2. Compute area and perimeter of triangle.
3. Compute area and perimeter of rectangle.
4. Compute area and perimeter of square.
5. Compute area of parallelogram.
6. Compute area and circumference of circle.
7. Compute periphery of ellipse and area of ellipse.
8. Compute the area of a circular segment.
9. Compute volume of cube.
10. Compute volume of rectangular solid.
11. Compute volume of cylinder.
12. Compute volume of sphere.

GEOMETRIC FORMULAS

UNIT VI

SUGGESTED ACTIVITIES

I. Provide students with objective sheet.

II. Provide students with information sheets.

III. Discuss unit and specific objectives.

IV. Discuss information sheets.

- A. Review basic algebra concepts as related to simple equations.
- B. Review basic calculator usage.
- C. Review exponents.
- D. Review basic geometric figures.

V. Pass out and discuss assignment sheets.

VI. Assist students as they work through assignment sheets.

VII. Give test.

VIII. Evaluate and record test results.

INSTRUCTIONAL MATERIALS

I. Included in the unit:

- A. Objective Sheet
- B. Information Sheet
- C. Assignment Sheets
  1. Assignment Sheet #1--Practical Problems Using Formulas
- D. Answers to Assignment Sheets
- E. Test
- F. Answers to Test

II. References

A. Anderson, John G. Technical Shop Mathematics New York, NY: Industrial Press, 1983.

B. Cleaves, Cheryl, Margie Hobbs, and Paul Dudenhafter. Basic Mathematics for Trades and Technologies, Englewood Cliffs, NJ: Prentice Hall, Incorporated, 1983.

C. Calter, Paul. Technical Math New York, NY: McGraw-Hill, Inc., 1979.

D. Goodson, C.E. and S.L. Miertschin. Technical Math with Applications. New York, NY: John Wiley and Sons, Inc., 1983.

E. Sparks, Fred W. Survey of Basic Mathematics New York, NY: McGraw-Hill, Inc., 1977.

GEOMETRIC FORMULAS

UNIT VI

INFORMATION SHEET

I. Terms and Definitions

A. Pythagorean Theorem--one of the most famous and useful procedures in mathematics; named for Pythagoras, a Greek mathematician; states that the square of the hypotenuse is equal to the sum of the square of the two legs of a right triangle; $c^2 = a^2 + b^2$ or $a^2 + b^2 = c^2$

(NOTE: The Pythagorean Theorem applies only to right triangles.)

B. Hypotenuse--the side of the triangle opposite the right angle

(NOTE: When working with the Pythagorean Theorem, one must know two sides of the right triangle in order to find the third side.)

C. Area--the number of square units on a flat surface

D. Perimeter--the distance around the outside of an object

E. Volume--the number of cubic units in a three-dimensional figure

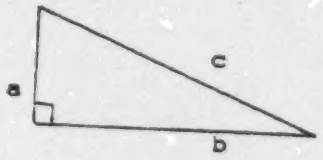
II. Procedures

A. Use of Pythagorean Theorem

Example:

Find side c (hypotenuse)

a = 3 m
b = 4 m
c = _____



$a^2 + b^2 = c^2$ formula

$3^2 + 4^2 = c^2$

$9 + 12 = c^2$

$25 = c^2$

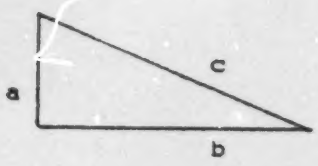
$\sqrt{25} = c$

$5 = c$

Example:

Find side b

a = 3 m
c = 5 m
b = _____



$a^2 + b^2 = c^2$ formula

$3^2 + b^2 = 5^2$

$9 + b^2 = 25$

$b^2 = 16$

$b = \sqrt{16}$

$b = 4$

B. Other Geometric Formulas

1. Area of a triangle--area = 1/2 base x height;
   a = 1/2 b x h

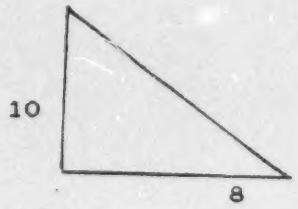
Example:

Area = 1/2 base x height

Area = 1/2 (10 x 8)

Area = 1/2 (80)

Area = 40 square units



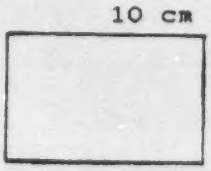
2. Area of a rectangle--area = length x width;
   a = l x w

Example:

Area = length x width

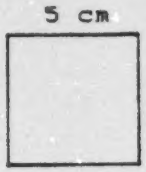
Area = 10 x 5

Area = 50 cm^2
(50 square centimeters)



3. Area of a square--area = side x side or side^2

Example:



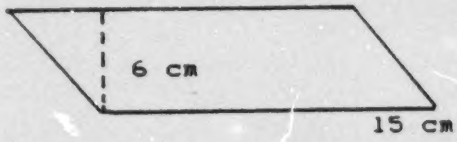
Area = side^2

Area = 5^2

Area = 25 cm^2
(25 square centimeters)

4. Area of a Parallelogram--area = base x height

Example:



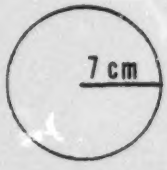
Area = base x height

Area = 15 x 6

Area = 90 cm^2
(90 square centimeters)

5. Area of a Circle--area = π x radius^2

Example:



Area = πr^2

Area = $(3.14)(r)^2$

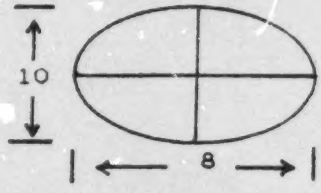
Area = 3.14 x 49

Area = 153.86 cm^2

(90 square centimeters)

6. Area of Ellipse--area = πab; a = 1/2 of major axis; b = 1/2 of minor axis

Example:



a = 1/2 (10) = 5

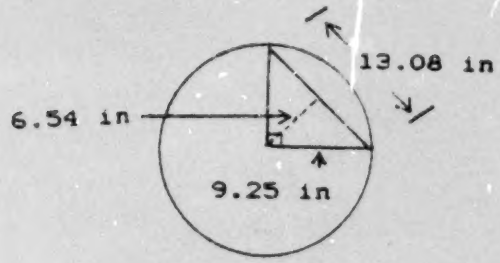
b = 1/2 (8) = 4

Area = πab

Area = (3.14)(5)(4)

Area = 62.80 square units

7. Area of a Segment



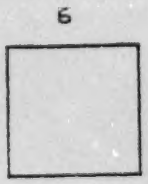
$$\text{area} = \frac{\text{degrees}}{360}\ \pi r^2 - 1/2 \text{ base x height}$$

$$\text{area} = \frac{90}{360}\ (3.14)(9.25)^2 - 1/2\ (13.08)(6.54)$$

area = 67.166 - 42.77

area = 24.39 sq. in.

8. Perimeter of a Square--perimeter = side (4)



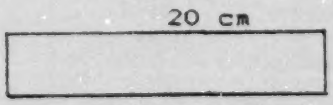
p = side (4)

p = 6 (4)

p = 24

9. Perimeter of a Rectangle

perimeter = 2 x length + 2 x width



p = 2l + 2w

p = 2(20) + 2(5)

p = 40 + 10

p = 50 cm

10. Perimeter of a Hexagon

perimeter = side (b)

p = 5 cm (6)

p = 30 cm

11. Circumference of a Circle

Circumference = π x Diameter

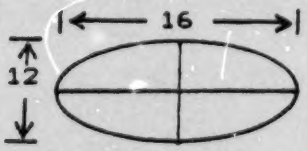
Circumference = π x D

C = 3.14 x 30

C = 94.20 cm^2

12. Periphery of an Ellipse

Perimeter = $\pi \sqrt{2a^2 + 2b^2}$



Find the periphery of an ellipse with axes of 16 cm and 12 cm. a = 1/2(16) = 8; b = 1/2(12) = 6

$P = \pi \sqrt{2a^2 + 2b^2}$

$P = 3.142 \sqrt{2(8)^2 + 2(6)^2}$

$P = 3.142 \sqrt{2(64) + 2(36)}$

$P = 3.142 \sqrt{128 + 72}$

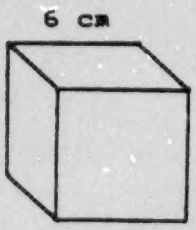
$P = 3.142 \sqrt{200}$

$P = 3.142 (14.14)$

$P = 44.4cm$

13. Volume of a Cube

Volume = Side ^{3}



Find the volume of a cube with 6 cm sides

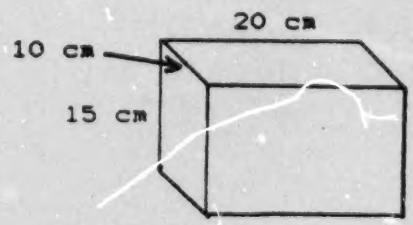
Volume = Side ^{3}

Volume = 6^3

Volume = 216 cm^3 (cubic centimeters)

14. Volume of a Rectangular Solid

Volume = length x width x height



V = lwh

V = 20 x 10 x 15

V = 3,000 cm^2

15. Volume of a Cylinder

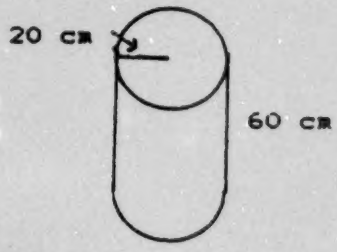
Volume = π x radius^2 x height

Compute the volume of a cylinder with a radius of 20 cm and a height of 60 cm.

Volume = $\pi r^2 h$

Volume = $(3.14)(20)^2)(60)$

Volume = 75,360 cm^2

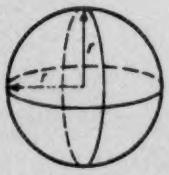


16. Volume of a Sphere

Compute the volume of a sphere with a radius of 5 in

Formula: $\frac{4\pi r^2}{3}$

r = 5 in.



$$V = \frac{4(3.142)(5)^3}{3}$$

$$V = \frac{4(3.142)(5)(125)}{3}$$

V = 523.7 cubic inches

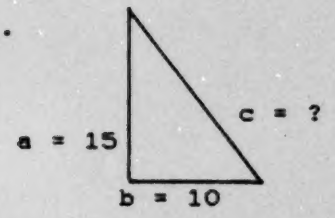
GEOMETRIC FORMULAS

UNIT VI

ASSIGNMENT SHEET #1--PRACTICAL PROBLEMS USING FORMULAS

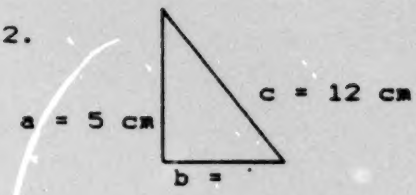
Directions: Work the following problems using the formulas provided in the information sheets. Use your calculator and π function. Round answers to nearest hundredth.

1.



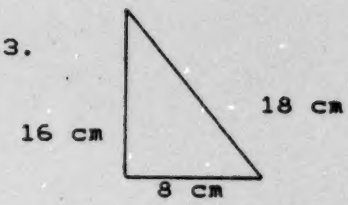
(a) Find side c ___cm
(b) Area of triangle = __cm^2
(c) Perimeter of triangle =__cm

2.



(a) Find side b ____cm
(b) Area of triangle = __cm^2
(c) Perimeter of triangle =__cm

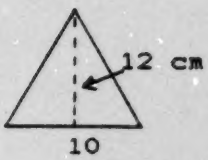
3.



(a) Area of triangle = __cm^2

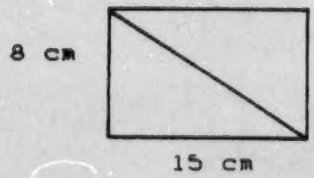
(c) Perimeter of triangle =__cm

4.



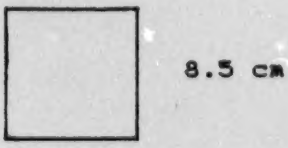
(a) Area = __cm^2

5.



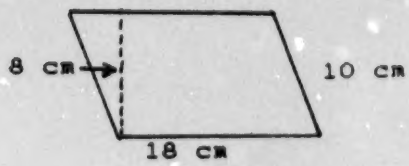
(a) Area of rectangle = __cm^2
(b) Perimeter of rectangle =__cm
(c) Length of diagonal line =__cm

6.



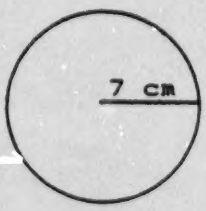
(a) Perimeter of square =__cm
(b) Area of square = __cm^2
(c) Length of diagonal line =__cm

7.



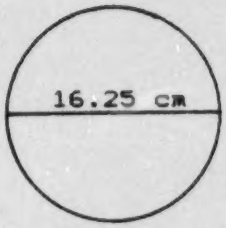
(a) Area of parallelogram = __cm^2
(b) Perimeter of parallelogram =__cm

8.



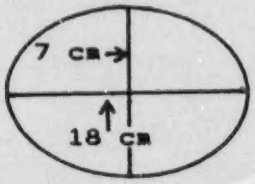
(a) Area of circle = __cm^2
(b) Circumference of circle =__cm

9.



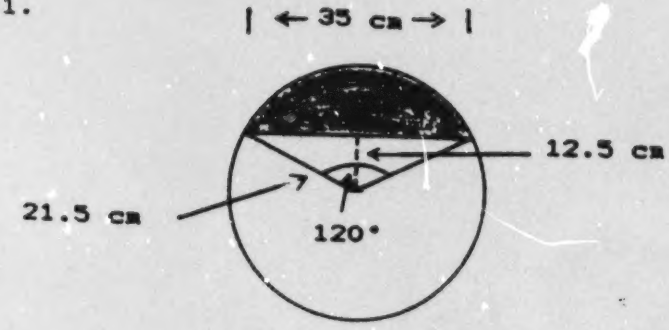
(a) Area of circle = __cm^2
(b) Circumference of circle =__cm

10.



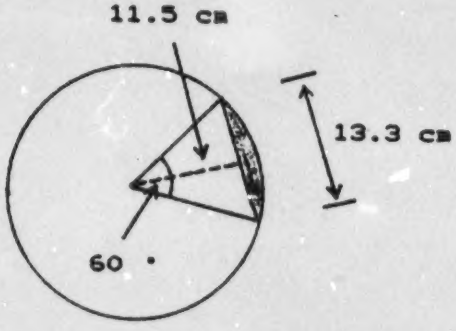
(a) Periphery of Ellipse =____cm

11.



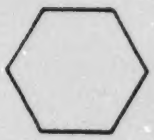
(a) Area of Shaded Segment =____cm^2

12.



(a) Area of Shaded Segment =____cm^2

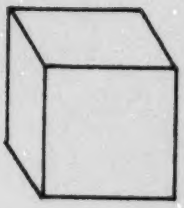
13. 8.5 cm



(a) Perimeter of Hexagon =____cm^2

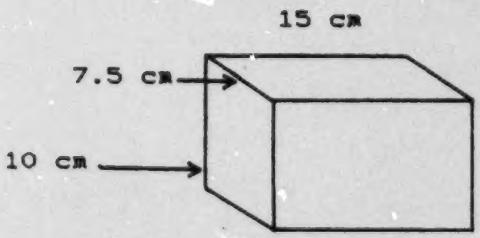
14.

15.5 cm



(a) Volume of Cube =____cm^3

15.



(a) Volume of Rectangular Solid =____cm^3

16.

radius = 7.5 cm

(a) Volume of Sphere = ____cm^3

17.

height = 25 cm

radius = 8.5 cm

(a) Volume of Cylinder = ____cm^3

16.

radius = 7.5 cm

(a) Volume of Sphere = ____cm^3

17.

height = 25 cm

radius = 8.5 cm

(a) Volume of Cylinder = ____cm^3

GEOMETRIC FORMULAS

UNIT VI

ANSWERS TO ASSIGNMENT SHEETS

Assignment Sheet #1

1. (a) $c = 18.2$ cm
   (b) 75 cm^2
   (c) 43 cm

2. (a) $b = 10.9$ cm
   (b) 27.25 cm^2
   (c) 27.9 cm

3. (a) 64 cm^2
   (b) 42 cm

4. $a = 60$ cm^2

5. (a) 120 cm^2
   (b) 46 cm
   (c) 17 cm

6. (a) 34 cm
   (b) 72.25 cm^2
   (c) 12.02 cm

7. (a) 144 cm^2
   (b) 56 cm

8. (a) 153

9. (a) 207.3 cm^2
   (b) 51.05 cm

10. (a) 85.80 cm

11. (a) 265.3 cm^2

12. (a) 16.14 cm^2

13. (a) 51 cm

14. (a) 3723.87 cm^3

15. (a) 1125 cm^3

16. (a) 1767.14 cm^3

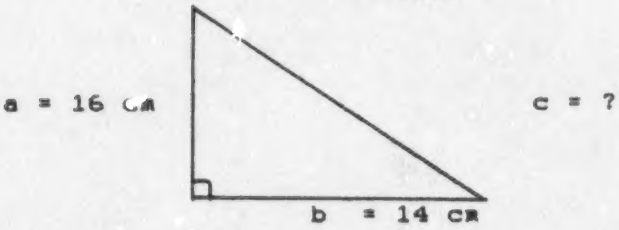
17. (a) 5674.50 cm^3

GEOMETRIC FORMULAS

UNIT VI

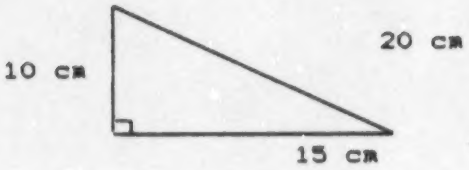
TEST

1.



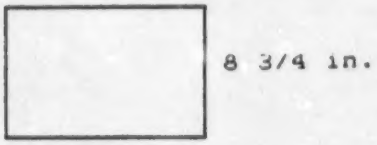
How many cm is line C? _____

2.



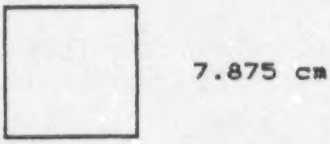
(a) Area of Triangle = _____
(b) Perimeter of Triangle = _____

3.



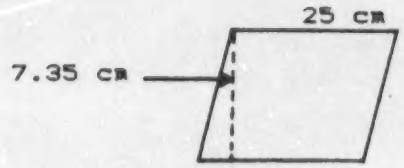
(a) Area of Rectangle = _____ sq. in.
(b) Perimeter of Rectangle = _____ in.

4.



(a) Area of Square = _____ sq. cm
(b) Perimeter of Square = _____ cm

5.

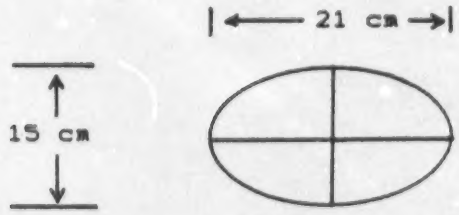


(a) Area of Parallelogram = _____ cm^2

6.

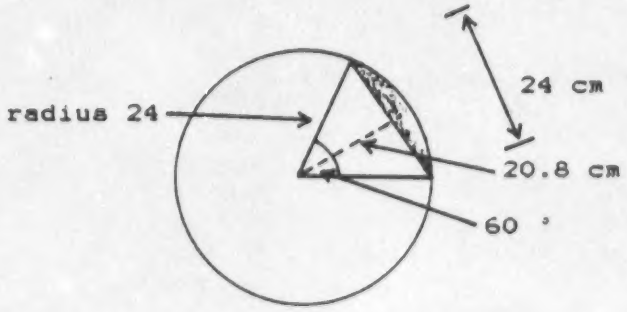
(a) Area of Circle = _____ cm^2
(b) Circumference of Circle = _____ cm

7.



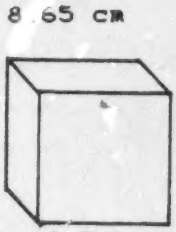
(a) Periphery of Ellipse = _____ cm
(b) Area of Ellipse = _____ cm

8.



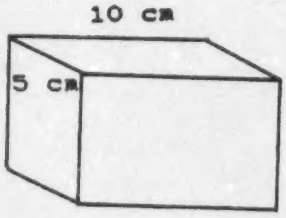
(a) Area of Shaded Segment = _____ cm

9.



(a) Volume of Cube = _____ cm^3

10.



(a) Volume of Rectangular Solid = _____ cm^3

11.

height = 25 cm

radius = 9.75 cm

(a) Volume of Cylinder = _____ cm^3

12.

radius 15.5 cm

(a) Volume of Sphere = _____ cm^3

GEOMETRIC FORMULAS

UNIT VI

ANSWERS TO TEST

1. 21.26 cm

2. a. 75 cm^2
   b. 45 cm

3. a. 62 57/64 sq. in.
   b. 31 7/8 in.

4. a. 62.01 cm^2
   b. 31.5 cm

5. 183.75 cm^2

6. a. 103.86 cm^2
   b. 36.12 cm

7. a. 57.32 cm
   b. 247.40 cm^2

8. 52 cm^2

9. 647.21 cm^3

10. 750 cm^3

11. 7466.19 cm^3

12. 15598.5 cm^3